Turgunbay Zhuraev,
PhD, assistant professor;

Zulaikho Tursunova,
Researcher,
Tashkent state pedagogical university

Absolute extensors of the class AE (0)

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Annotation. *In this article studied non-compact analogies topological spaces at the class* $AE(0)$. **MSC**: 54B15, 54B30, 54B35, 54C15, 54C60, 54D30

1. Introduction

In this note we consider the spaces of Milyutin, Dugundji, Michael, and investigate geometric, topological properties under the action of certain functors in the category $Tych-$ of spaces and continuous mappings into itself. The concept of the Dugundji compactum, introduced by A.Pelchinsky (1), turned out to be very fruitful and led to the creation of important new methods in general topology. Answering a question from Pelchinsky, R. Haydon showed (2) that every Dugundji compact is dyadic (3). continuous image of the generalized Cantor discontinuity D^{τ}. On the other hand, the Dugundji compacta are exactly the compact sets of the class $AE(0)$. The theory $AE(0)$ of compacta was extended by AN. Dranishnikov (4) to absolute extensors in dimension n. Also in this paper, non-compact analogies of the Dugundji space, the Milyutin space, and Michael are defined. Their topological properties and geometric properties are studied using some covariant functors. Terminology and designation, not explained below, are the same as in the books (1,3,5).

2. The main part

For a Tikhonov space X we denote by $C(X)$ the space of continuous functions defined on X with a compact-open topology.

The linear operator $u: C(X) \to C(Y)$ is said to be regular if the following conditions are satisfied:

a) the mapping $u: C(X) \to C(Y)$ is continuous;

b) if $\varphi \geq 0$, then $u(\varphi) \geq 0$ (that is, the operator is positive);

c) $u(1_X) = 1_Y$, where $1_X : X \to \{1\} \subset R$ is a constant function. those. the operator takes constant functions to constants. Every continuous mapping $f: X \to Y$ generates a regular operator $f^*: C(Y) \to C(Y)$ by the formula $f^*(\varphi) = \varphi \circ f$.

If X is closed in Y and for every $\varphi \in C(X)$ the restriction of the function $u(\varphi)$ to X coincides with φ, then the operator is called the extension operator.

If the mapping $f:X \to Y$ is surjective, then the regular operator $u:C(X) \to C(Y)$ is called the regular averaging operator.

Definition (4). $R-$a compact space X (or, $R-$compact) is called a Dugundji space if any $C-$ embedding $f:X \to Y$ into the Tikhonov space Y, has a regular extension operator $u:C(X) \to C(Y)$.

Definition (1). A perfect epimorphism $f:X \to Y$ is called Milyutin if it admits a regular averaging operator $u:C(X) \to C(Y)$.

A Tikhonov space X is called a Milyutin space if there exists a Milutin epimorphism $f:N^{\tau} \to X$, where $N-$is the set of natural numbers.

We call an embedding R^A in I^A, where A is an arbitrary index set, standard if for any $B \subset A$, the following relations are satisfied: $t_B(R^A) = R^B$ and $t_B(R^A) = \pi_B$, where through $t_B : I^A \to I^B$ and $\pi_B : R^A \to R^B$ - the corresponding design.

Theorem 1. If X is a Dugundji space. Then X is a $AE(o)$ space.

Evidence. Let τ denote R the weight of the space X. If $R-\omega(X) = \omega$, then X is closed in R^{ω}. Consequently, X is a Polish space. By Proposition 1.1.4 in (5), the space X is $AE(o)$ is a space.

Suppose that $X-R-$is compact and $C-$ is embedded in R^A and $u:C(X) \to C(R^A)$ is a regular extension operator. In this case, the space X is the inverse limit of the factorizing strictly ω spectrum $S_X = \{X_B, P_C^B, \exp_{\omega} A\}$. We denote by $A(\alpha)$ the subsets of the index set A whose ordinal $\alpha < \tau$. The mapping $P_\alpha : X \to X_\alpha$ is defined as a shallow $t_{A(\alpha)}\big|_X = P_\alpha$ and $P^{\beta}{}_{\alpha} : X_\beta \to X_\alpha$ is the restriction of the mapping $P_{C(\alpha)}^{B(\alpha)} = P_C^B$.

Now let the systems $A(\alpha)$ having the following conditions (properties) be constructed:

I. $A(o)$ is a point;

II. If γ is the limit ordinal $< \tau$, then $A(\gamma) = \bigcup\limits_{\alpha<\gamma} A(\alpha)$;

III. For each α, the difference $A(\alpha+1) \setminus A(\alpha) -$ is countable;

IV. For each α and for all $f \in C(X_\alpha)$ the map $u(f \circ P_\alpha)$ is consistent with the map $(f \circ \pi_{A(\alpha)})$ on the subspace $X_\alpha \times R^{A \setminus A(\alpha)}$ space R^A;

V. For all α and any $f \in C(X_{\alpha+1})$ constraint and $(f \circ P_{\alpha+1})\big|_{X_\alpha \times R^{A\setminus A(\alpha)}}$ is a factor of the through mapping $\pi_{A(\alpha+1)}$.

From condition iii) it follows that the map (neighboring projections) $P_{\alpha+1}^{\alpha}$ has a Polish kernel, i.e. we have the following diagram

$$\begin{array}{ccc} X_{\alpha+1} & \xrightarrow{i} & X_\alpha \times R^{A(\alpha+1)\setminus A(\alpha)} \\ \downarrow {\scriptstyle P_\alpha^{\alpha+1}} & & \downarrow {\scriptstyle \pi_\alpha} \\ & X_\alpha & \end{array}$$

By virtue of conditions (V), we define a regular extension operator $u_{\alpha+1}:C(X_{\alpha+1})\to C(X_\alpha\times R^{A(\alpha+1)\setminus A(\alpha)})$ Polojaya $u_{\alpha+1}(f)\circ\pi_{A(\alpha+1)}=u(f\circ P_{\alpha+1})$, where $f\in C(X_{\alpha+1})$. It follows from property (IV) that the equality $u_{\alpha+1}(f\circ P_\alpha^{\alpha+1})=f\circ\pi_\alpha$,
where $f\in C(X_\alpha)$. It is obvious that the map $\pi_\alpha:X_\alpha\times R^{A(\alpha+1)\setminus A(\alpha)}\to X_\alpha$ - is open.
Let us prove that the map $P_\alpha^{\alpha+1}:X_{\alpha+1}\to X_\alpha$ is also open.
Evidence. In this case we have the following diagram

$$\delta\left(X_{\alpha+1}\right)\subset P_\beta\left(X_{\alpha+1}\right)$$

$$\therefore\quad 4\ \delta\downarrow 3\quad \square\quad \theta\quad \uparrow\aleph$$

$$X\xrightarrow{P_{\alpha+1}}X_{\alpha+1}\xrightarrow{i}X_\alpha\times R^{A(\alpha+1)\setminus A(\alpha)}$$

$$\square_{P_\alpha}\quad 1\downarrow P_\alpha^{\alpha+1}\ 2\quad \square_{\pi_\alpha}$$

$$X_\alpha$$

$$A(\alpha+1)\setminus A(\alpha)=A(\omega)$$

Where $\aleph:X_\alpha\times R^{A(\omega)}\to P_\beta(X_{\alpha+1})$ the continuous map generated by the regular extension operator

$$u_{\alpha+1}:C(X_{\alpha+1})\to C(X_\alpha\times R^{A(\omega)}),$$

$$\theta=\aleph\big|_{i(X_{\alpha+1})}:i(X_{\alpha+1})\to\delta(X_{\alpha+1}),i(X_{\alpha+1})\subset X_\alpha\times R^{A(\omega)}$$

Let $x_{\alpha+1}^o\in X_{\alpha+1}$, $P_\alpha^{\alpha+1}(x_{\alpha+1}^o)=x^o{}_\alpha$. Since the diagram (1) is commutative, there exists a point $x^o\in X$ such that $P_{\alpha+1}(x^o)=x^o{}_\alpha$, $P_\alpha(x_0)=x^0{}_\alpha$ it is known that for each $\alpha\in A$, the space X_α is a Polish space. Consider a sequence of points $x_\alpha^n\in X_\alpha$ converging to the point x_α^o i. $\lim\limits_{n\to\infty}\rho_\alpha(x_\alpha^n;x_\alpha^o)=0$, where ρ_α is the metric in X_α
We show that there exists a sequence of points $x_{\alpha+1}^n$ in the space $X_{\alpha+1}$ converging to the point $x_{\alpha+1}^o$ such that $P_\alpha^{\alpha+1}(x_{\alpha+1}^n)=x_\alpha^n$.
Put $B_n(x_{\alpha+1}^0,n)=\{x\in X_{\alpha+1}:\rho_{\alpha+1}(x,x_{\alpha+1}^0)\le\frac{1}{n}\}$

$$x_\alpha^n\times R^{A(\omega)}\cap i(X_{\alpha+1})=A(x_{\alpha+1})$$

$$\delta\circ\theta(A(x_{\alpha+1}^n))\cap i^{-1}(A(x_{\alpha+1}^n))=C(x_{\alpha+1}^n)$$

$$B_n(x_{\alpha+1}^0,n)\cap C(x_{\alpha+1}^n)=D(x_{\alpha+1}^n).$$

It is obvious that for each $n\in N$ the set $D\left(x_{\alpha+1}^n\right)$ is nonempty. Now for each $n\in N$ we choose from the points $x_{\alpha+1}^n\in D(x^n{}_{\alpha+1})$ so that the diagrams (2), (3), (4) are commutative.
The sequence of points $x^1{}_{\alpha+1},x^2{}_{\alpha+1},\ldots,x^n{}_{\alpha+1},\ldots$ converges to the point $x^o{}_{\alpha+1}$ and $P_\alpha^{\alpha+1}(x_{\alpha+1}^n)=x_\alpha^n$. Hence, the map $P_\alpha^{\alpha+1}:X_{\alpha+1}\to X_\alpha$ is open.
We construct a system of sets $A(\alpha)$ by transfinite induction.

Let $A(0)=\varnothing$, then X_0 - is a point.

The family $(f_\zeta)_{\zeta<\tau}$ in $C(X)$ separates the points of X from the set.

Assume that the sets $A(\alpha)$ are defined for all ordinals $\beta<\tau$ and satisfy conditions (ii), (iii), (iv) and (v). Let ζ be the first ordinal for which f_ζ is not a factorization of the through mapping P_β. By Theorem 6.27 (or Corollary 6.28 in (3)), for a function f_ζ defined on R^A there exists a countable subset of $C\subset A$ such that $f_\zeta = g\circ P_C$ i.e. f_ζ - depends on a countable number of coordinates or f is a factor of the through mapping P_C.

In what follows, in the same way as in the proof of Theorem 3 in (2), we choose the index set B.

Now we define $A(\beta+1)=A(\beta)\cup B$. We have the following diagram

$$\begin{array}{ccc} X & \rightarrow & X_\beta \times R^{A\setminus A(\beta)} \\ \quad_{P_\beta} & & _{t_{A(\beta)}} \\ & X_\beta & \end{array} \qquad (2)$$

The regular extension operator $\upsilon : C(X)\rightarrow C\left(X_\beta\times R^{A\setminus A(\beta)}\right)$ is defined by setting $\upsilon(f)=u(f)|_{X_\beta\times R^{A\setminus A(\beta)}}$. Since for $\alpha=\beta$ we put $\upsilon(f\circ P_\alpha)=f\circ\pi_{A(\beta)},\ f\in C(X_\beta)$.

notice, that

a) for each α, the space $X_{\alpha+1}$. C-embedded in $X_\alpha\times R^A$ and has a regular operator $u_{\alpha+1}$ extensions (figure 1). Note that $X_{\alpha+1}$ Polish space. Therefore, $X_{\alpha+1}$ is the Dugundji space and $X_{\alpha+1}\in AE(0)$;

b) The spectrum of the $S_X=\{X_\alpha,\, P_{\alpha\beta},\, \alpha\}$ is completely ordered and ω is complete, the neighboring projections $P_\alpha^{\alpha+1}$ are open and have a Polish kernel;

c) X_0 - is the Polish $AE(0)$ space;

d) By virtue of the openness of the neighboring projections $P_\alpha^{\alpha+1}$ by Corollary 3.3.27 (5) $P_\alpha^{\alpha+1}$ soft.

e) By the compactness of the space X by the theorem of 3.2.17 in (5), the spectrum of the S_X factorizing strict X spectrum is R.

f) $\lim S_X -$ is homeomorphic to the space X.

Now, by Proposition 3.5.4. (5) the space X is the $AE(0)$ space. Theorem 1 is proved.

Theorem 2. The class of Michael spaces coincides with the class of Dugundji spaces.

Evidence. Let X be the Michael space and $\psi : N^A \rightarrow X$ an invertible surjection. Let T be a zero-dimensional $R-$compact space, $S-$is closed in T and $\phi : S\rightarrow X$

$$\begin{array}{ccc} N^{A} & \xrightarrow{\psi} & X \to R^{\tau}, \quad X - C\text{-invested in } R^{\tau} \\ \theta'\uparrow & & \theta \uparrow \phi \\ T & \xrightarrow[i]{} & S \end{array}$$

In view of the 0-invertibility of the map ψ, there exists $\theta : S \to N^{A}$ such that $\psi \circ \theta = \phi$. It is obvious that $N^{A} \in AE(0)$. By virtue of the zero-dimensionality of the compactum T and $N^{A} \in AE(0)$ there exists a mapping $\theta^{'} : T \to N^{A}$ such that $\theta^{'} \circ i = \theta$. Consider the mapping $\overline{\theta} = \psi \circ \theta^{'} : T \to X$. The mapping $\overline{\theta}$ is an extension of the map ϕ i.e $\overline{\theta}|_{S} = \phi$ i.e $X \in AE(0)$. Hence, X is the Dugundji space.

The converse statement is obvious, as each $X \in AE(0)$ space is a 0-soft image of some power N^{τ}. (zero soft maps are 0-invertible). Theorem 2 is proved.

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