DOI 10.12851/EESJ201701C02ART03

Shaylovbek A. Saipnazarov,
ScD, researcher;

Malika T. Ortikova,
researcher,
Tashkent University of economics;

Concept and Mechanism of Integration Algebraic and Geometric Methods

Key words: *example, concept, trigonometric method, method equations, method volume, method of identical, mechanism, algebraic, geometric.*

Annotation: *This article provides examples of the concept and mechanism of the algebraic and geometric methods.*

To determine the concept of the integration of algebraic and geometric methods, you need to the structure of each of these methods.

Any mathematical method (algebraic or geometric) in the process of learning mathematics we understand as a way to cognitive activity, and any activity is techniques. For example, the method of equations and method systems of equations consists of the following methods: reception, based on the preparation and solving equations or systems of equations.

Similarly, each geometric method consists of different techniques based on the use of metric the ratio of triangles, etc.

Mechanism of the integration of algebraic and geometric methods in the case of orientation "from the algebra to the geometry of" is as follows: in algebraic field is given problem solved the two methods, at first it is solved algebraic method, then on the border of two fields is her translation on the geometric language and already geometric field it is solved geometric the method of. Similarly, is the process and the case "of geometry to algebra" such a way to integration we called a combination of methods.

The mechanism of integration can consist of three possible situations:

Situation 1. Given algebraic problem, it is translated on the geometric language, and the resulting geometric the problem is solved geometric the method of.

Example 1. To solve the system of equations

$$\begin{cases} x = \sqrt{z^2 - 2} + \sqrt{y^2 - 2} \\ y = \sqrt{x^2 - 3} + \sqrt{z^2 - 3} \\ z = \sqrt{y^2 - 4} + \sqrt{x^2 - 4} \end{cases}$$

The solution. Suppose that x, y, z are the sides of a triangle, height of which are equal to $\sqrt{2}, \sqrt{3}$ and 2, and the triangle should be obtuse. To find the x, y, z use the fact that a triangle whose sides are inversely proportional to the heights of this, similar to the triangle

$$x=\frac{1}{2\sqrt{2}S},\quad y=\frac{1}{2\sqrt{3}S},\quad z=\frac{1}{4S} \qquad (1)$$

where $S=\sqrt{p\left(p-\frac{1}{\sqrt{2}}\right)\left(p-\frac{1}{\sqrt{3}}\right)\left(p-\frac{1}{2}\right)},\quad 2p=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{2}$

this formula to calculate the value is difficult, so we apply the theorem of cosines, then calculate the sine of the angle between the two sides of a triangle. First find the cosine of the angle between the parties $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{3}}$

$$\cos\alpha=\frac{\frac{1}{2}+\frac{1}{3}-\frac{1}{4}}{2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{3}}}=\frac{7\sqrt{6}}{24},\qquad \sin\alpha=\sqrt{1-\left(\frac{7\sqrt{6}}{24}\right)^2}=\frac{\sqrt{282}}{24}$$

$$S=\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{282}}{24}=\frac{1}{2}\cdot\frac{1}{\sqrt{6}}\cdot\frac{\sqrt{6\cdot\sqrt{47}}}{24}=\frac{\sqrt{47}}{48} \qquad (2)$$

From formula (1) and (2) we find the desired variables x, y and z.

$$x=\frac{1}{2\sqrt{2}\cdot\frac{\sqrt{47}}{48}}=\frac{24}{\sqrt{2}\cdot\sqrt{47}}=12\sqrt{\frac{2}{47}}$$

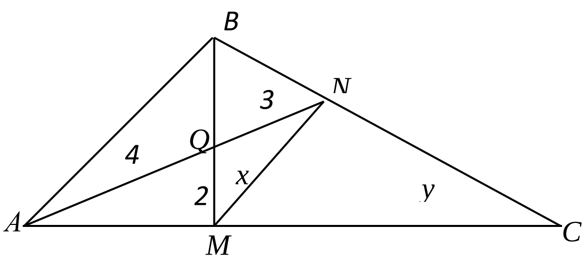
$$y=\frac{1}{2\sqrt{3}\cdot\frac{\sqrt{47}}{48}}=\frac{24}{\sqrt{3}\cdot\sqrt{47}}=8\sqrt{\frac{3}{47}}$$

$$z=\frac{1}{4\cdot\frac{\sqrt{47}}{48}}=\frac{12}{\sqrt{47}}=12\sqrt{\frac{1}{47}}.$$

Situation 2. Specified geometric problem, it is translated on the algebraic language and received an algebraic the method of.

Example 2. In the triangle ABC on the side AC taken point M, and on the side of N. Segments AN and BM intersect at the point Q. Find the area of the triangle CMN, if the area of triangles QMA, QAB and QBN respectively, are equal to 2, 4 and 3 (Figure 1).

The solution. We denote by x the area of the triangle QMN, through the y area of the triangle CMN, then



$$\frac{QN}{QA}=\frac{x}{2}=\frac{3}{4}$$

Figure 1.

$$\frac{AM}{MC}=\frac{2+x}{y}=\frac{2+4}{3+x+y}$$

Hence. We obtain the algebraic system of equations

$$\begin{cases}\frac{x}{2}=\frac{3}{4}\\ \frac{2+x}{y}=\frac{2+4}{3+x+y}\end{cases} \Rightarrow \begin{cases}x=1,5\\ \frac{2+1,5}{y}=\frac{6}{3+1,5+y}\end{cases}$$

$$\frac{3,5}{y}=\frac{6}{4,5+y}, \quad 6y=15,75+3,5y, \quad 2,5y=15,75, \qquad y=6,3.$$

Situation 3. Given integrated task. (Term “integrated task” will be called the task at decision which is necessary to take advantage of two or more methods (algebraic and geometric)). Part of it is solved algebraic method, as part of geometric may repeated alternation of these methods in the process of solutions. All actions in this case are carried out in the integration. A number of these problems are many stereometrics task for which it is necessary to use knowledge of geometry, algebra and trigonometry. We give an example of one of them.

Example 3. Sphere inscribed in a right circular cone with the angle α at the vertex axial section. It is entered in this field with the same cone vertex angle axial section. Find the angle α if the ratio of the volume of the first cone to a second cone is equal to 8.

The solution. Denote the radius of the sphere in terms of R and consider the axial section of each of the cones. Second cone can be arranged in an arbitrary manner within the sphere. We arrange it so that the forming of both cones are parallel. Express the radii reason cones through the R. In the triangle FOK angles OFK and OKF equal to $\frac{\alpha}{2}$. Consequently, the angle EOK is equal to their sum, that. le. α . Of the triangle EOK we find $EK=R\sin\alpha$ (Figure 2).

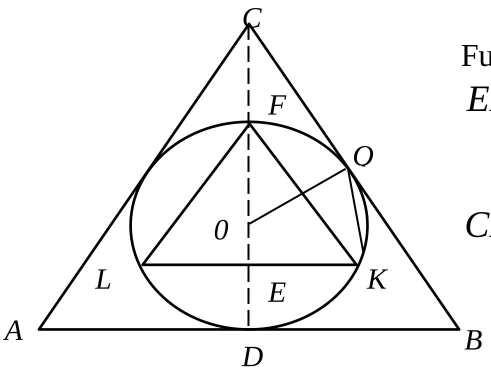


Figure 2.

Further,

$$EF=EO+OF=R(1+\cos\alpha) \quad (3)$$

$$CD=CO+OD=R\left(1+\frac{1}{\sin\frac{\alpha}{2}}\right) \quad (4)$$

$$DB = DC \cdot tg\frac{\alpha}{2} = R\left(1+\frac{1}{\sin\frac{\alpha}{2}}\right)\cdot tg\frac{\alpha}{2} = R\cdot\frac{1+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \qquad (5)$$

Now we can define the volume cones. Use of the formula (3), (4) and (5) get the volume of cones

$$V_1 = \frac{1}{3}\pi DB^2 CD = \frac{1}{3}\pi\left(R\cdot\frac{1+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\right)^2\cdot R\left(1+\frac{1}{\sin\frac{\alpha}{2}}\right) =$$

$$= \frac{1}{3}\pi R^3\frac{\left(1+\sin\frac{\alpha}{2}\right)^2}{\cos^2\frac{\alpha}{2}}\cdot\frac{1+\sin\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} = \frac{1}{3}\pi R^3\frac{\left(1+\sin\frac{\alpha}{2}\right)^3}{\cos^2\frac{\alpha}{2}\sin\frac{\alpha}{2}}$$

$$V_2 = \frac{1}{3}\pi EK^2\cdot EF = \frac{1}{3}\pi\left(R\sin\alpha\right)^2\cdot R\left(1+\cos\alpha\right) = \frac{1}{3}\pi R^3\sin^2\alpha\cdot 2\cos^2\frac{\alpha}{2} =$$

$$= \frac{1}{3}\pi R^3\cdot 4\sin^2\frac{\alpha}{2}\cdot\cos^2\frac{\alpha}{2} = \frac{1}{3}\pi R^3\cdot 8\sin^2\frac{\alpha}{2}\cdot\cos^4\frac{\alpha}{2}$$

Set op now the ratio of the volume and equate it to 8. After the simple changes will come to the equation with respect to α :

$$1+\sin\frac{\alpha}{2} = 4\sin\frac{\alpha}{2}\cdot\cos^2\frac{\alpha}{2} \quad \text{or} \quad 1+\sin\frac{\alpha}{2} = 4\sin\frac{\alpha}{2}\cdot\left(1-\sin^2\frac{\alpha}{2}\right)$$

since the $1+\sin\frac{\alpha}{2}\neq 0$ (otherwise there is no cane).

Divide both sides by $1+\sin\frac{\alpha}{2}$

$$4\sin^2\frac{\alpha}{2} - 4\sin\frac{\alpha}{2} + 1 = 0$$

$$\left(2\sin\frac{\alpha}{2}-1\right)^2 = 0,$$

$$2\sin\frac{\alpha}{2} - 1 = 0,$$

$$\sin\frac{\alpha}{2}=\frac{1}{2}$$

from whence $\alpha = 30^\circ$.

When solving this problem have been involved in the main following methods:

1) Trigonometric method; 2) method equations; 3) method volume; 4) method of identical transformations.