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Forms and Means of Integration Algebraic and Geometric Methods

Key words: *geometric solution, algebraic and geometric methods, coordinate methods, integration.*

Annotation. *This article provides forms and means the integration of algebraic and geometric methods.*

The initial form of integration serves a lot of algebraic and geometric methods for solving the same problem. The following form of integration is ordering when and this set of algebraic and geometric methods appears order relation between individual methods that gives an additional unifying a sign for outside the set of unite methods. At the same time students is given task, solve the problem of different methods and indicates the of their application of. For example, to solve the problem at the beginning algebraic and then geometric methods or to solve geometric task trigonometric method based on the theorem cosines, then vector and coordinate methods.

Here are examples of the integration of algebraic and geometric methods of different direction.

Example 1.

Point *L* lies on the side of *AC* triangle *ABC*. Segment *BL* crosses the median *CM* at the point 0. That the area of the triangle *BMO* is equal to 3, and the area quadrangle *AMOL* is equal to 4. Find the area of the triangle *ABC*.

The solution. 1-stage (translation tasks on the algebraic language).

We denote the area of triangles *A0L*, *A0M*, *L0C* and *B0C*, respectively s_1, s_2, s_3 and s_4 . Then on the condition of the problem $s_1 + s_2 = 4$ *OM* for the triangle *A0B* median, so $s_2 = 3$ of triangles *A0L* and *L0C*

$$\frac{1}{s_3} = \frac{AL}{LC}$$

of triangles *ABL* and *BLC*

$$\frac{3 + s_1 + s_2}{s_3 + s_4} = \frac{AL}{LC}$$

Finally we get following algebraic systems of equations

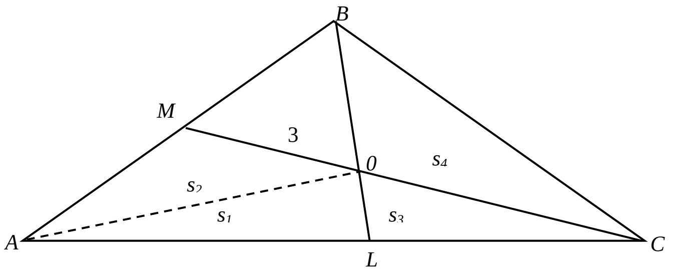


Figure -1

$$\begin{cases} s_1 + s_2 = 4 \\ s_2 = 3 \\ \dfrac{7}{s_3 + s_4} = \dfrac{1}{s_3} \\ 4 + s_3 = 3 + s_4 \end{cases}$$

2-stage (solution of the problem on the algebraic language)

Solving the system of equations we find

$$s_1 = 1,\ \ s_2 = 3,\ s_3 = 0,2,\ s_4 = 1,2$$
$$S_{ABC} = 7 + 0,2 + 1,2 = 8,4$$

This example shows us geometric method (method areas) and algebraic method (method equations and method of identical transformations).

We show the same connection with the decision algebraic tasks.

Example 2.

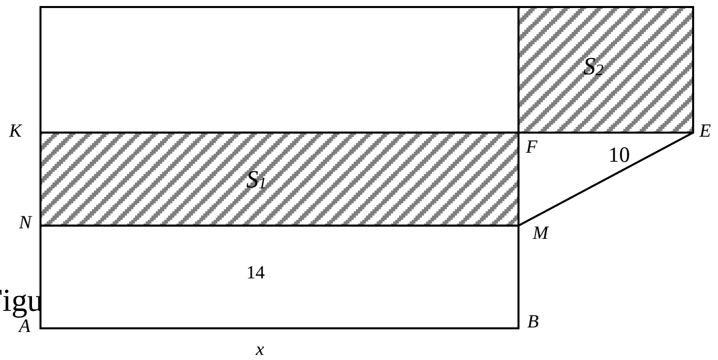
Train had to go through 54 km. Passing 14 km, he was arrested in semaphore 8 min. Increasing rate after that, by 10 km/hear, he arrived at the destination scheduled. To determine the initial speed train.

Solving this problem with the help of a two-dimensional charts. Two-dimensional charts this is square one or more rectangles (triangles, parallelograms), side and the Heights of which represent the numerical values considered values, and the area of the rectangle (triangles, parallelograms) depicts their product.

The solution. 1-stage (the construction of a two-dimensional chart).

Let $AB = x$ depicts speed trains scheduled (km/h), *AD* – the time of his motion (hour) (figure – 2).

Then S_{ABCD} (S_{ABCD} - the area quadrangle) determines the way which must pass train (km), $S_{ABCD} = AB \cdot AD = 54$



Figu

Let $S_{ABMN} = 14$ (S_{ABMN} - the area quadrangle), and S_1 defines the way, which took place to train during the parking. $FE = 10$, then $KE = x + 10$ - increased speed trains.

2-stage (solution of the resulting geometric tasks).

Due to increase the speed of 10 km/h train caught lost time.

Therefore $s_1 = s_2$, but $s_1 = \frac{2}{15}x,\ s_2 = 10 \cdot PE$, and $PE = \frac{S_{KEPD}}{x+10} = \frac{40}{x+10}$,

then we obtain the equation

$\frac{x}{15} = \frac{200}{x+10}$ or $x^2 + 10x - 3000 = 0$, location $x_1 = 50, x_2 = -60$

The second root of the equation not satisfy, so $AB = 50$

3-stage (translation answer with geometric language on the natural).

The answer: the initial speed train 50 km/h.

In the examples 1 and 2 shows the one relationship between the geometric and algebraic methods. Increase of relationships leads to a new high – quality form of integration, which is called the system. System – well organized a lot of forming holistic unity. System supports the most perfect shape of the synthesis of United components. An example of this form the integration of methods serves as a method for solving, includes as components algebraic and geometric methods. We give an example of the tasks which leads to the decision of the system of equations.

Example – 3.

In figure 3 $\angle ABC = \angle ADB = 90^0$ and $AB = AP$. Prove that *BP* bisects $\angle CBD$.

The solution.

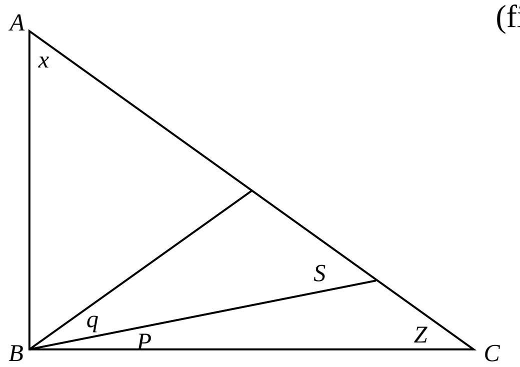


Figure -3

We introduce the notation for all considered angles (figure – 3), then we obtaion the following equality:

$$\begin{cases} q + z = s \\ p + y = s, \\ x + z = 90^0 \\ x + y = 90^0 \end{cases}$$

Performing conversion, we obtain that $y = z$ and $p = q$, that qed.

Up to date the geometry of students formed system knowledge about the area, of such triangles, on the circle, of vectors.

Ance with this in solving problems method is used area as, the method of such triangles, the method of circles, vector method.

Up to date algebra students learn square equations and inequalities, quadratic function its properties and schedule. Therefore, there is an opportunity to integrate the method of space and method of similarity, vector method, the method, of trigonometry the method of parallel direct.

Give examples of.

Example – 4.

The area of rectangular triangle is equal to 30 and the length of hypotenuse is equal to 13. Get the cathetuses.

Solution of the problem results in a system of equations the second degree:

$$\begin{cases} \frac{1}{2}xy = 30 \\ x^2 + y^2 = 169 \end{cases}$$

were's x and y the length of cathetuses.

Example – 5.

Two of the triangle are equal accordingly 6 and 8. Median carried out to parties are perpendicular.

(figure – 4). Get the area of the triangle.

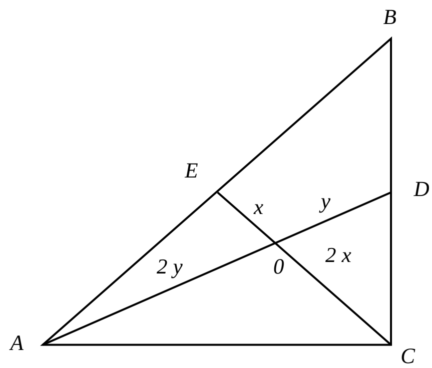


Figure – 4

Taking $OE = x, OD = y$ and carrying out the translation of the problem on the algebraic language, come to the system of equations:

$$\begin{cases} x^2 + 4y^2 = 16, \\ 4x^2 + y^2 = 9 \end{cases}$$

Example – 6. To solve the positive numbers system of equations

$$x^2 + xy + y^2 = 25,$$
$$y^2 + yz + z^2 = 49,$$
$$z^2 + zx + x^2 = 121$$

Geometric solution. Let us consider three segment $OA = x, OB = y, OC = z$, form the pairwise angles 120 degrees (figure – 5). By theorem cosines on the basis of the set of equations we get $AB = 5, BC = 7, CA = 11$. But in the triangle ABC angle B greater than 120 degrees (check). Therefore, the system has no positive solutions.

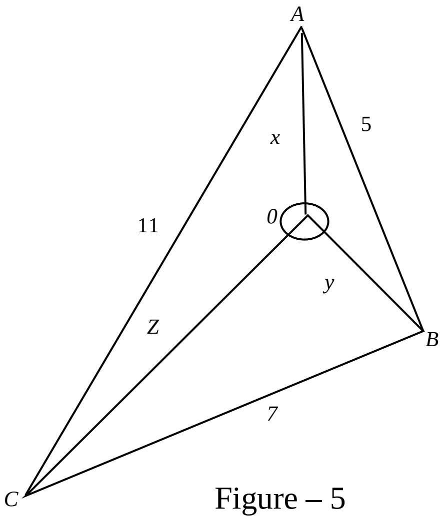


Figure – 5

Algebraic solution. Of the first two equations, we find that $x^2 < 25,\ z^2 < 49$, and then

$$x^2 + zx + x^2 < 49 + 35 + 25 = 109\ ,$$

and therefore, positive numbers in this system has no solutions.