DOI 10.12851/EESJ201508C02ART01

Mikhail G. Kolonutov,
ScPh, chief engineer,
Novgorodaudit-Energo

Electrodynamics as the Mechanics of Electric Field

Key words: *Electric field, continuum mechanics, electrodynamics, quantization of energy.*

Annotation: *The article describes an experiment where continuum mechanics has been applied to the electric field. An equation has been derived, the solutions of which are identical to the basic laws (postulates) of electrodynamics. The causes have been determined that explain the appearance of corpuscular properties of radiation that cause photoelectric effect.*

"The highest goal of natural science is to reduce any phenomenon to motion which, in turn, is subject to description by means of theoretical mechanics."
Gustav Robert Kirchhoff, 1865

Introduction

The proposed article is based on the standpoint that the description of the processes occurring in the electric field may be constructed on the theoretical grounding of continuum mechanics. There will be no contradiction with physical laws in the assumption that the electric field, as every material substance that is able to convey the action of one body upon another, possesses the quality of elasticity (in the mechanical sense of the term). It will be evident from the forthcoming exposition that with full certainty this assumption may be transferred to the category of assertions, because the basic laws of electrodynamics, such as Faraday's law of electromagnetic induction or Maxwell's postulates, may be obtained by applying the methods of theoretical mechanics to the electric field. Just like other laws of the theory of electricity, they can leave the category of postulates and move on to the category of conclusions based on logical analysis and proof.

The paper explores the processes going on in the electric field in case of different kinds of motion of charge carriers: uniform motion, uniformly accelerated motion and oscillatory motion. For the case of oscillatory motion, the mechanism of wave process initiation is shown, wave equation is formulated and its solutions are obtained. The results of the research allow making a considerable headway in the understanding of the physics of wave processes going on in the electric field, for instance, in explaining the reasons for the appearance of corpuscular properties in electromagnetic radiation, which the existing electrodynamics is unable to explain.

1 The equation of the electrodynamic state of the electric field

The equation of the electric field disturbance wave that arises in case of motion of a charge carrier can be derived most easily for the electric field of a cylindrical capacitor. Let us assume there is a capacitor whose inner and outer plate radii equal r_0 and R respectively. The charge at the plates is distributed evenly with the linear density λ=const. In course of the analysis we will be using the cylindrical coordinate system (r, θ, z), whose applicate axis coincides with the longitudinal axis of the capacitor.

As is generally known, the density of electrostatic field energy of such capacitor w is expressed in terms of relation (1.1),

$$w = \frac{\lambda^2}{8\,\pi^2\varepsilon_0 r^2}\,. \tag{1.1}$$

The field density ρ will be determined by using the Einstein relation,

$$\rho = \frac{w}{c^2} = \frac{\lambda^2}{8\,\pi^2\varepsilon_0 r^2 c^2}\,. \tag{1.2}$$

Let us pick out an elementary field fragment in the shape of a ring with inner radius r, sectional width and height Δr and Δl respectively, whose section is shown in Figure 1. The mass of the field forming the ring will make up Δm,

$$\Delta m = \rho\,\Delta V = \frac{\lambda^2}{8\,\pi^2\varepsilon_0 r^2 c^2} 2\pi r \Delta r \Delta l \tag{1.3}$$

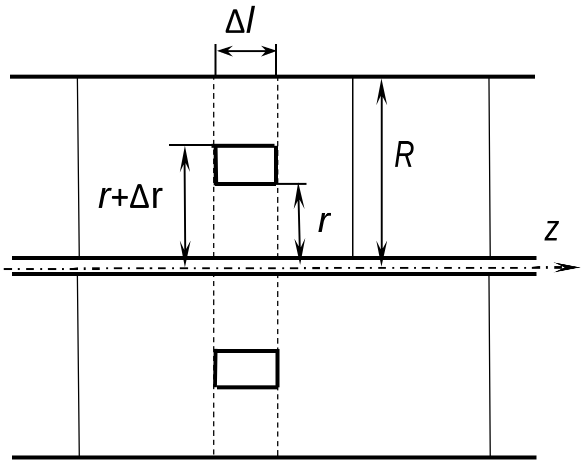

Figure 1

If the inner plate is set to longitudinal accelerated motion, than the displacement in the axial direction of certain points of the field located at various distances from the plate, will be different. For this reason, the capacitor field will undergo shear deformation, which will bring about the change in the shape of field lines, and the ring will assume the shape which is shown in Figure 2.

It is natural to think that in case of shear deformation, mechanical stresses spring up in the electric field, under the influence of which the disturbance brought about by the motion of the inner plate propagates through the electric field in the direction of the outer plate in shear (transverse) waves. As is known, propagation speed of such waves is connected with the shear modulus G in the medium where waves propagate, by the relation, c = $(G/\rho)^{1/2}$ where ρ is the density of the medium, i.e. the density of the electric field.

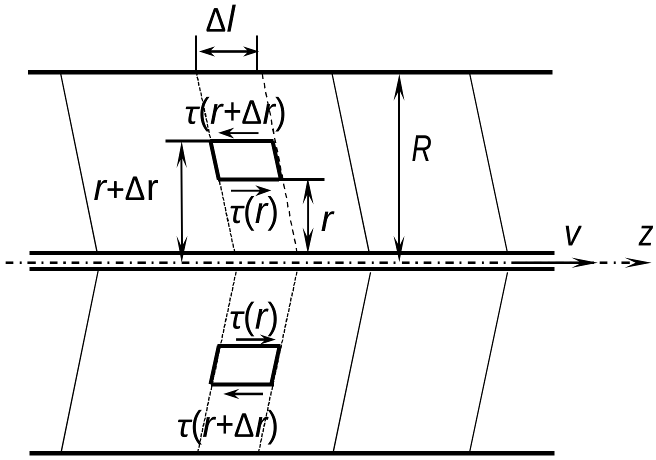

Figure 2

Assuming that the disturbance propagation speed in the electric field equals the speed of light, let us use the relation for determining the elastic modulus of the electric field,

$$G = c^2\rho = \frac{\lambda^2}{8\,\pi^2\varepsilon_0 r^2} = w. \tag{1.4}$$

Shear deformation in the theory of elasticity is determined by the derivative of displacement u along the radius, $\gamma = \partial u/\partial r$, therefore the mechanical stress tangents τ(r) and τ(r+Δr) shown in Figure 2 may be expressed through formulae (1.5) and (1.6),

$$\tau(r,t) = G\gamma = w\frac{\partial u(r,t)}{\partial r}, \tag{1.5}$$

$$\tau(r+\Delta r,t) = \tau(r,t) + \frac{\partial\tau}{\partial r}\Delta r. \tag{1.6}$$

Stresses (1.5) and (1.6) call forth the forces acting on the inner and outer surfaces of the ring. The first of them, $f_1(r,t)$, is caused by the tangential stress τ(r,t),

$$f_1(r,t) = \tau(r,t)2\pi r\Delta l\,. \tag{1.7}$$

The second, $f_2(r+\Delta r,t)$, is caused accordingly by the stress τ(r+Δr,t),

$$f_2(r+\Delta r,t) = \tau(r+\Delta r,t)2\pi(r+\Delta r)\Delta l. \tag{1.8}$$

Neglecting the second order infinitesimal summands, let us find resultant force Δf, that sets the ring to motion,

$$\Delta f = f_2 - f_1 = \left(\frac{\partial\tau(r,t)}{\partial r} + \frac{\tau(r,t)}{r}\right)2\pi r\Delta r\Delta l\,. \tag{1.9}$$

The resultant Δf (1.9), ring mass Δm (1.3) and acceleration $a = \partial^2 u/\partial t^2$ are connected by Newton's second law; therefore, considering relation (1.5) we obtain equation (1.10),

$$\frac{\partial^2 u}{\partial r^2} + \left(\frac{\partial w}{w\,\partial r} + \frac{1}{r}\right)\frac{\partial u}{\partial r} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}\ . \tag{1.10}$$

Working out the parenthesis, we arrive at the required equation of motion in displacements u,

$$\frac{\partial^2 u}{\partial r^2} - \frac{1}{r}\frac{\partial u}{\partial r} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}\,. \tag{1.11}$$

This wave equation may by right be called the equation of the electrodynamic state of the electric field, since its solutions constitute theoretical foundation for a multitude of experimentally discovered laws of the contemporary theory of electricity.

2 Solutions to the equation of the electrodynamic state of the electric field

2.1 Uniform motion of charge carrier

For solving equation (1.11) let us use the method of separation of variables. Assuming that displacement u is the product of two functions $R(r)$ and $T(t)$, $u = R(r)T(t)$, then substituting the product into equation we will obtain

$$\frac{1}{R}\frac{d^2R}{dr^2} - \frac{1}{Rr}\frac{dR}{dr} = \frac{1}{Tc^2}\frac{d^2T}{dt^2} = k, \tag{2.1}$$

where parameter k can take negative and positive values or be equal to zero.

Now suppose that $k = 0$, then from (2.1) we will get two equations

$$\frac{d^2R}{dr^2} - \frac{1}{r}\frac{dR}{dr} = 0, \tag{2.2}$$

$$\frac{d^2T}{dt^2} = 0. \tag{2.3}$$

The respective solutions to equations (2.2) and (2.3) have the form

$$R = C_1 + C_2 r^2, \tag{2.4}$$

$$T = C_3 + C_4 t. \tag{2.5}$$

From the physical meaning of the problem, it follows that displacement cannot be the quadratic function of distance r. To satisfy this condition, the constant of integration C_2 should be put equal to zero, $C_2 = 0$. Then we get linear dependence of displacement from time, which is characteristic of uniform motion

$$u = RT = C_1C_3 + C_1C_4t. \tag{2.6}$$

Product $u_0 = C_1C_3$ specifies the initial position, product $v = C_1C_4$ specifies the of uniform motion rate of charge carrier.

Thus, case $k = 0$ corresponds to uniform motion of charge carrier:

$$u = u_0 + v\,t. \tag{2.7}$$

Deformation of the field in this case equals to zero, $\partial u/\partial r = 0$. Absence of deformation of the field in case of uniform motion of charge carrier indicates that the field lines remain perpendicular to the longitudinal axis of the linear charge carrier regardless of its motion rate, which makes it clear that the laws of electrostatics are the same in any inertial system.

2.2 Uniformly accelerated motion of charge carrier

In case of uniformly accelerated motion, equation (1.11) takes the form of (2.8)

$$\frac{\partial^2 u}{\partial r^2} - \frac{1}{r}\frac{\partial u}{\partial r} = \frac{a}{c^2}\,, \tag{2.8}$$

where a is the acceleration of motion.

The solution to the problem is function (2.9)

$$u = \frac{ar^2}{2c^2}\ln r - \frac{ar^2}{4c^2} + \frac{r^2}{2}C_1(t) + C_2(t), \qquad (2.9)$$

where $C_1(t), C_2(t)$ are arbitrary functions of time. To determine the so far unknown function $C_1(t)$ we will have to know the deformation $\gamma = \partial u/\partial r$, so let us move on to finding it.

Figure 3 illustrates the resultant of tangential forces which is applied to the inner surface of the ring f_1, and the force f which is equal to it but has a different direction, acting upon the field region which is adjacent to the ring in a radial direction.

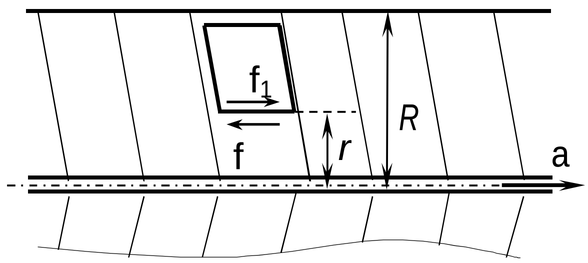

Figure 3

From Newton's second law, it follows that

$$\mathbf{f}_1 = -\mathbf{f} = m\mathbf{a} = \frac{\mathbf{a}\, l\, \lambda^2}{4\pi\varepsilon_0 c^2}\ln\frac{R}{r}, \qquad (2.10)$$

where m is the mass of the ring-shaped fragment of the field.

By differentiating relation (2.10) with respect to l and λ we will determine the force that corresponds to the unit length of the field fragment and unit charge density. In the theory of electricity, the force acting upon a unit charge is called electric field intensity, thus

$$\mathbf{E}_{ed} = \frac{d\mathbf{f}}{dl\, d\lambda} = -\frac{\mathbf{a}\,\lambda}{2\pi\varepsilon_0 c^2}\ln\frac{R}{r}. \qquad (2.11)$$

The intensity $\mathbf{E}_{ed}$ characterizes a special electric field that appears only as a result of accelerated motion, so let us call such field an electrodynamic field and its intensity will be called electrodynamic intensity. Relation (2.11) shows that the vector of electrodynamic intensity $\mathbf{E}_{ed}$ is always in the reverse direction from that of the acceleration vector $\mathbf{a}$.

Dividing the module of force f by surface area we will find tangential stress

$$\tau = \frac{f}{2\pi rl} = -\frac{a\,\lambda^2}{8\pi^2 r\varepsilon_0 c^2}\ln\frac{R}{r}, \qquad (2.12)$$

which in accordance with relation (1.5) may be expressed in terms of deformation $\gamma = \partial u/\partial r$. Comparing (1.5) and (2.12) we obtain:

$$\tau = w\frac{\partial u}{\partial r} = -\frac{a\,\lambda^2}{8\pi^2 r\varepsilon_0 c^2}\ln\frac{R}{r}, \qquad (2.13)$$

whence it follows that

$$\gamma = \frac{\partial u}{\partial r} = -\frac{ar}{c^2}\ln\frac{R}{r}. \qquad (2.14)$$

Using (2.14) let us express electrodynamic field intensity (2.11) in terms of deformation and electrostatic field intensityE_{es}. It is not difficult to verify that

$$E_{ed} = \gamma E_{es} = -\frac{a\,\lambda}{2\pi\varepsilon_0 c^2}\ln\frac{R}{r}. \qquad (2.15)$$

Expression (2.15) permits treating electric field deformation as a change in the picture of lines of force in comparison with the similar picture for the electrostatic case. Qualitatively, the deformation is pictured in Figure 4, where dashed lines denote the electrostatic field lines. Deformation is formally represented by the tangent of angle α between vector $\mathbf{E}_{es}$ and sum vector $\mathbf{E}=\mathbf{E}_{es}+\mathbf{E}_{ed}$, $\gamma = \tan\alpha$.

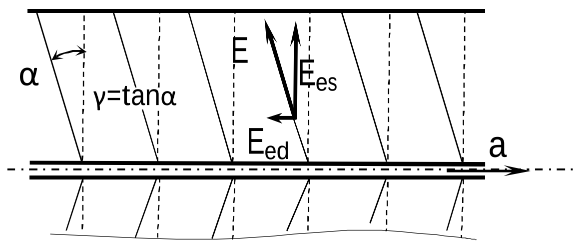

Figure 4

Now let us come back to determining the function of time $C_1(t)$ in expression (2.9). Equating the deformation defined through the derivative of expression (2.9), and the deformation given by formula (2.14):

$$\gamma = \frac{\partial u}{\partial r} = \frac{ar}{c^2}\ln r + rC_1(t) = -\frac{ar}{c^2}\ln\frac{R}{r}, \qquad (2.16)$$

whence we obtain that function $C_1(t)$ at a given acceleration is a constant:

$$C_1 = -\frac{a}{c^2}\ln R. \qquad (2.17)$$

Substituting C_1 in expression (2.9) let us determine function $C_2(t)$ by using boundary condition $u(r_0, t) = at^2/2$ (uniformly accelerated motion). As a result, we will get the required axial displacement as the function of time and radial distance,

$$u = \frac{at^2}{2} - \frac{ar^2}{2c^2}\left(\ln\frac{R}{r} + \frac{1}{2}\right) + \frac{ar_0^2}{2c^2}\left(\ln\frac{R}{r_0} + \frac{1}{2}\right). \qquad (2.18)$$

Field deformation under this kind of motion was defined above.

2.3 Oscillatory motion of charge carrier

In expression (2.1), when the quantity k is negative, $k< 0$, the method of separation of variables yields

$$R(r) = C_1 r J_1\left(\frac{\omega r}{c}\right) + C_2 r Y_1\left(\frac{\omega r}{c}\right), \qquad (2.19)$$

$$T(t) = C_3\sin(\omega t) + C_4\cos(\omega t), \qquad (2.20)$$

where C_1, C_2, C_3, C_4 are constants of integration; $J_1(z)$ is the first kind first order Bessel function, $Y_1(z)$ is second kind first order Bessel function. The general solution of equation (1.11) is the product $R(r) \cdot T(t)$:

$$u = C_1 C_4 r J_1\left(\frac{\omega r}{c}\right)\cos(\omega t) + C_2 C_3 r Y_1\left(\frac{\omega r}{c}\right)\sin(\omega t) +$$
$$+ C_1 C_3 r J_1\left(\frac{\omega r}{c}\right)\sin\omega t + C_2 C_4 r Y_1\left(\frac{\omega r}{c}\right)\cos\omega t. \quad (2.21)$$

Let the initial phase of charge carrier oscillations be equal to zero. This condition, by virtue of the fact that $J_1(0) = 0$ and $\sin(0) = 0$, will be satisfied by the function, formed by the first two summands of expression (2.21):

$$u = C_1 C_4 r J_1\left(\frac{\omega r}{c}\right)\cos(\omega t) + C_2 C_3 r Y_1\left(\frac{\omega r}{c}\right)\sin(\omega t). \quad (2.22)$$

Relation (2.22) represents the traveling wave only in case when $C_1C_4 = C_2C_3$, so, marking $C = C_1C_4 = C_2C_3$, from the last relation we have

$$u = Cr\left(J_1\left(\frac{\omega r}{c}\right)\cos(\omega t) + Y_1\left(\frac{\omega r}{c}\right)\sin(\omega t)\right). \quad (2.23)$$

For defining the constant of integration C let us determine the speed v_z of longitudinal (axial) movement of the field by differentiating relation (2.23) with respect to time,

$$v_z = \frac{\partial u}{\partial t} = C\omega r\left(-J_1\left(\frac{\omega r}{c}\right)\sin(\omega t) + Y_1\left(\frac{\omega r}{c}\right)\cos(\omega t)\right). \quad (2.24)$$

Let us assume that the movement of charge carrier is of harmonic character, then the electric current induced by this motion may be expressed by the relation

$$i(t) = \lambda v_z(r_0, t) = I_m \cos(\omega t), \quad (2.25)$$

whence it follows that

$$v_z(r_0, t) = I_m \cos(\omega t)/\lambda\,. \quad (2.26)$$

Now equating expressions (2.24) and (2.26) at the instant of time $t = 0$ at $r = r_0$, we will find the constant of integration C,

$$C = \frac{I_m}{\lambda\,\omega\, r_0\, Y_1(\omega r_0/c)}. \quad (2.27)$$

Knowing the constant of integration, we will obtain the final form of the solution to the differential equation

$$u = \frac{I_m}{\lambda\,\omega\, Y_1(\omega r_0/c)}\frac{r}{r_0}\left(J_1\left(\omega\frac{r}{c}\right)\cos(\omega t) + Y_1\left(\omega\frac{r}{c}\right)\sin(\omega t)\right). \quad (2.28)$$

The derivative of displacement (2.28) with respect to distance r represents shear deformation:

$$\gamma = \frac{\partial u}{\partial r} = \frac{I_m}{\lambda\ c\, Y_1(\omega r_0/c)}\frac{r}{r_0}\left(J_0\left(\omega\frac{r}{c}\right)\cos\omega t + Y_0\left(\omega\frac{r}{c}\right)\sin\omega t\right). \quad (2.29)$$

Figure 5 presents the r-u plot of displacement function (2.28) at the following parameter values: $c = 3 \cdot 10^8$ m/s; $r_0 = 1$ mm; $t = 2$ s; $Im/\lambda = 0{,}006$ m/s; $\omega = 6{,}28 \cdot 10^6$ s^{-1} The plot reflects the shape of the line of force which it takes at oscillatory motions of charge carrier with the frequency of 1 MHz.

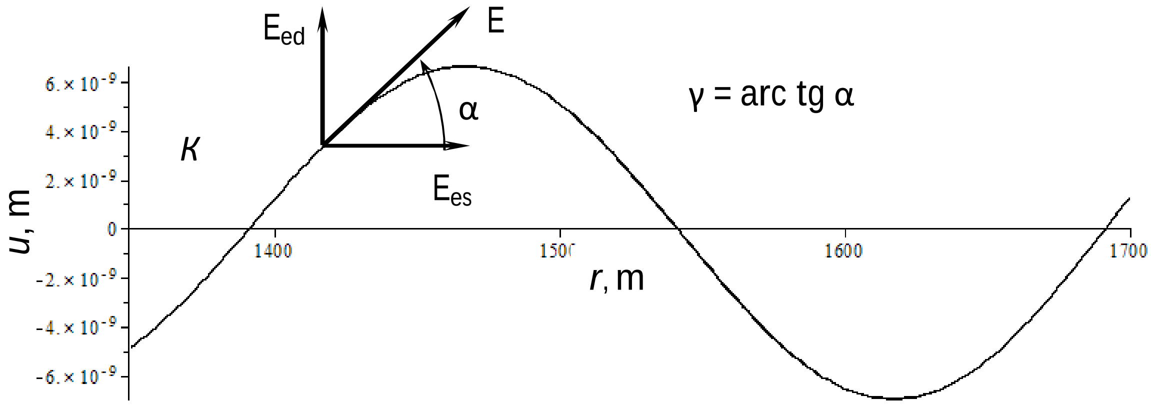

Figure 5

The same Figure 5 qualitatively shows a vector diagram of the intensity of the electrostatic, electrodynamic and resultant fields at a certain spatial point *K*. The resultant intensity has two components: radial electrostatic $\mathbf{E}_{es}$ and longitudinal electrodynamic $\mathbf{E}_{ed}$, caused by acceleration. The vector diagram demonstrates that the electrodynamic component of intensity may be found as the product of shear deformation and electrostatic field intensity:

$$E_{ed} = \gamma E_{es} = \frac{I_m}{2\pi\varepsilon_0 r_0 c\, Y_1(\omega r_0/c)}\left(J_0\left(\omega\frac{r}{c}\right)\cos(\omega t) + Y_0\left(\omega\frac{r}{c}\right)\sin(\omega t)\right). \quad (2.30)$$

Function (2.30) describes the traveling wave of electrodynamic field intensity. At such distance *r* that makes approximation of Bessel functions possible, we will get the familiar form of this wave which is expressed in terms of harmonic function,

$$E_{ed} = \frac{I_m}{\sqrt{2\pi r\omega c}\,\pi\,\varepsilon_0 r_0 Y_1(\omega r_0/c)}\cos\left(\omega t + \frac{\pi}{4} - \omega\frac{r}{c}\right). \qquad (2.31)$$

All the above concerned the processes at work in the electric field of a cylindrical capacitor. But with a big enough value of the outer plate radius, $R\mapsto\infty$, it is easy to move on to the description of the processes going on in the moving field of a single-isolated linear charge carrier, or to representing the processes in the field of a current-carrying conductor.

A conductor may always be imagined consisting of two charged linear charge carriers, one of which is the ionic lattice of the conductor material, and the other corresponds to electron gas. Linear charge densities at these carriers are the same but have opposite signs. Therefore, at any point in space, the vector sum of electrostatic intensities of these charge carriers will always equal zero, unlike the intensity developed by accelerated motion of electrons, that always takes place when the conductor is carrying alternating current. The intensity $\mathbf{E}_{ed}$ of the electrodynamic field of electrons always remains uncompensated and can easily be detected experimentally. This very phenomenon underlies the operating principle of transformers, whose performance at present fails to find proper theoretical grounding, since the law of electromagnetic induction which has traditionally been offered for explanation, is still the generalization of experimental data, not the result of a theoretical conclusion.

3 The law of electromagnetic induction

To confirm that the outlined viewpoint is adequate, let us find, by using relation (2.16), the induced emf that arises in some closed circuit which is placed in the same plane as the conductor carrying linearly swelling current $i = \lambda(v_0 + at)$ (Figure 6).

The induced emf at the sides AC and BD which are perpendicular to the vector of intensity $\mathbf{E}_{ed}$ will be equal to zero, whereas at the sides AB and CD which are parallel to this vector, the induced emf in accordance with (2.16) will make

$$e_{AB} = l\,E_{ed}(r_1) = -\frac{l\,a\,\lambda}{2\pi\varepsilon_0 c^2}\ln\frac{R}{r_1}, \qquad (3.1)$$

$$e_{CD} = l\,E_{ed}(r_2) = -\frac{l\,a\,\lambda}{2\pi\varepsilon_0 c^2}\ln\frac{R}{r_2}, \qquad (3.2)$$

where a is the acceleration experienced by those charge carriers whose motion generates the electric current, λ is the linear density of charge carriers.

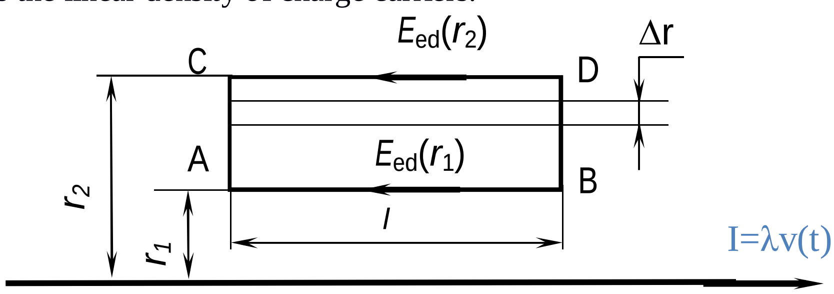

Figure 6

Summing algebraically the induced emf of the sides in accordance with Kirchhoff's voltage law, we will obtain

$$e = e_{AB} - e_{CD} = -\frac{l\,a\,\lambda}{2\pi\varepsilon_0 c^2}\ln\frac{r_2}{r_1}. \qquad (3.3)$$

For comparison, let us find the same quantity on the grounds of the law of electromagnetic induction.

The induction of the magnetic field developing around a current-carrying conductor is expressed by formula (3.4)

$$B = \frac{i(t)}{2\pi\varepsilon_0 c^2 r} = \frac{v(t)\lambda}{2\pi\varepsilon_0 c^2 r} \qquad (3.4)$$

The magnetic flux ΔΦ through an elementary surface of length l and width Δr will equal $\Delta\Phi = B\Delta r l$. Integrating this expression over r from r_1 to r_2 , we will find the magnetic flux Φ through the whole surface enclosed by the ACDB contour,

$$\Phi = \frac{l\,v(t)\,\lambda}{2\pi\varepsilon_0 c^2}\ln\frac{r_2}{r_1} \qquad (3.5)$$

Now taking the derivative of the flux Φ with respect to time, we will find the required expression for the induced emf,

$$e = -\frac{d\Phi}{dt} = -\frac{l\,a\,\lambda}{2\pi\varepsilon_0 c^2}\ln\frac{r_2}{r_1} \tag{3.6}$$

Relation (3.6) obtained by using Faraday's law, coincides with expression (3.3), which is the result of the analysis of the electric field as an object of the theory of elasticity.

In case of oscillatory motion of charge carrier, which corresponds to alternating current in the conductor, the induced emf that develops in the ABDC contour, is expressed by the relation

$$e = lE_{ed}(r_1) - lE_{ed}(r_2), \tag{3.7}$$

where $E_{ed}(r)$ is calculated by means of formula (2.34). Determining the same quantity using Faraday's law we will arrive at

$$e_\Phi = \frac{l\,\omega\,\lambda\,v_m}{2\,\pi\,\varepsilon_0 c^2}\ln\frac{r_2}{r_1}\sin(\omega t). \tag{3.8}$$

Figure 7 illustrates that the induced emf (3.8) at distances well beyond the wavelength, virtually coincides with the induced emf (3.7); in the figure these induced emfs are plotted in relation to time.

The solid line represents the plot corresponding to relation (3.7), whereas the line formed by circles represents the plot of the induced emf according to Faraday (3.8). Both the plots are constructed at the following parameter values: ω = 314 c^{-1}, $I_m = \lambda \cdot v_m$=10 A, r_0=10^{-3}m, r_1=0.1m, r_2=0.2m.

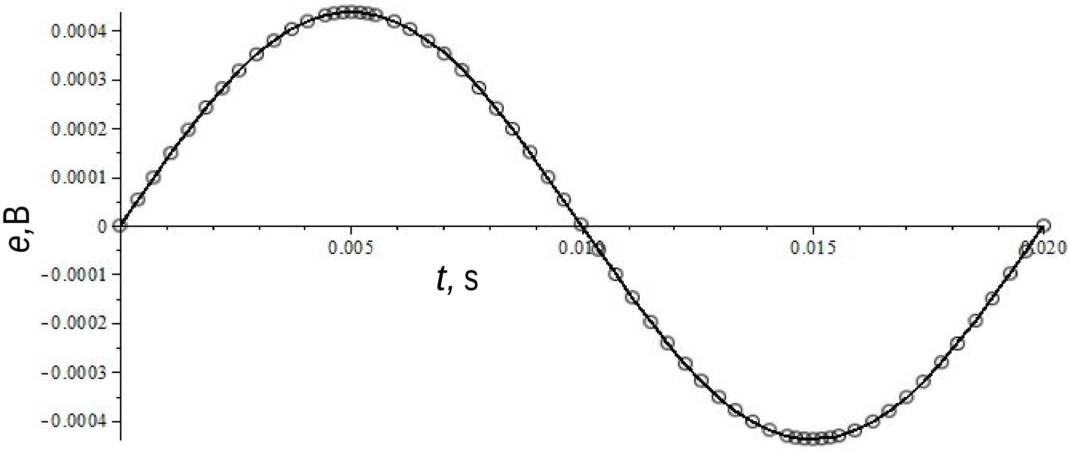

Figure 7

Comparison of formulae based on Faraday's law, and those derived in accordance the outlined approach, points to the fact that they do not differ in their predictive validity. But there is a fundamental difference between them, which lies in the fact that the formulae suggested herein are grounded on the basic concepts of continuum mechanics, whereas the classical, traditionally used formulae were only based on the generalization of experimental data effected by Michael Faraday.

4 Maxwell's equations

As was noted in the introduction, Maxwell's equations underlying the contemporary electromagnetic theory may be derived by applying the methods of the theory of elasticity to the electric field. Let us show by way of transformations that Maxwell's equations can be brought to equation (1.11) which was obtained in exactly this way.

This is the way Maxwell's equations look:

$$\text{rot}\,\mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \tag{4.1}$$

$$\text{rot}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{4.2}$$

In case of longitudinal motion of linear charge carrier that was analyzed above, the electric field intensity is the sum of electrostatic and electrodynamic field intensities, $E = E_{es} + E_{ed}$. The electrostatic component of intensity does not depend on time, that is why the right-hand side of equation (4.1) will only contain the derivative of the electrodynamic component of the electric field intensity. The right-hand side of this equation will then assume the form (4.3),

$$\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \varepsilon_0 \frac{\partial \mathbf{E}_{ed}}{\partial t} = \varepsilon_0 E_{es} \frac{\partial \gamma}{\partial t} \mathbf{1}_\text{z} = \varepsilon_0 E_{es} \frac{\partial^2 u}{\partial r\, \partial t} \mathbf{1}_\text{z}, \tag{4.3}$$

where $\mathbf{1}_z$ is the unit vector of the applicate axis.

Let us move on to rearranging the left-hand side of Maxwell's first equation. As is well known, magnetic field intensity is expressed by the relation

$$\mathbf{H} = \varepsilon_0 \boldsymbol{V} \times \mathbf{E}_{\text{эс}} = \varepsilon_0 E_{es} (\partial u/\partial t) \mathbf{1}_\theta\,. \tag{4.4}$$

Considering (4.4), the left-hand side of equation (4.1) takes the form that coincides fully with the right-hand side of the equation. Indeed, since $dE_{es}/dr = -E_{es}/r$, we have

$$\text{rot}\mathbf{H} = \left(\frac{\partial H}{\partial r} + \frac{H}{r}\right)\mathbf{1}_z = \varepsilon_0 \left(\frac{\partial E_{es}}{\partial r}\frac{\partial u}{\partial t} + E_{es}\frac{\partial^2 u}{\partial r \partial t} + \frac{E_{es}}{r}\frac{\partial u}{\partial t}\right)\mathbf{1}_z$$

$$= \varepsilon_0 E_{es} \frac{\partial^2 u}{\partial r \partial t} \mathbf{1}_z. \qquad (4.5)$$

Thus, the right-hand side of equation (4.1) turned out to be equal to its left-hand side. Maxwell's first equation turned out to be an identity, hence no worthwhile information can be extracted from it. Let us move on to examining Maxwell's second equation.

The rotor of the electrostatic component of intensity $\mathbf{E}_{es}$ equals zero, and since $E_{ed} = E_{es}\, \partial u/\partial r$, then the left-hand side of equation (4.2) will be rearranged as follows:

$$\text{rot}\,\mathbf{E} = -\left(\frac{dE_{es}}{dr}\frac{\partial u}{\partial r} + E_{es}\frac{\partial^2 u}{\partial r^2}\right)\mathbf{1}_\theta = -\left(-\frac{E_{es}}{r}\frac{\partial u}{\partial r} + E_{es}\frac{\partial^2 u}{\partial r^2}\right)\mathbf{1}_\theta. \tag{4.6}$$

Considering (4.6), equation (4.2) assumes the form of (4.7)

$$-\left(-\frac{E_{es}}{r}\frac{\partial u}{\partial r} + E_{es}\frac{\partial^2 u}{\partial r^2}\right)\mathbf{1}_\theta = -\mu_0 \varepsilon_0 E_{es}\frac{\partial^2 u}{\partial t^2}\mathbf{1}_\theta\,. \tag{4.7}$$

Dividing both parts of the last expression by ($-E_{es}$), we will arrive at equation (4.8) which coincides absolutely with equation (1.11):

$$\frac{\partial^2 u}{\partial r^2} - \frac{1}{r}\frac{\partial u}{\partial r} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}\,. \tag{4.8}$$

Therefore, the conducted analysis has shown that equation (1.11) of the electrodynamic state of the electric field and Maxwell's second equation may be converted into each other, which means that their solutions cannot be contradictory.

But the problem with Maxwell's equations lies in the fact that they recognize as material substance not only the electric but also the magnetic field. This leads to treating functions $E(r,t)$ and $H(r,t)$ as arguments on equal rights in Maxwell's equations. In reality, these functions are complex functions of the form $E[u(r,t)]$ and $H[u(r,t),\partial u/\partial t]$. Trying to find the solutions of Maxwell's equation in the form of E(r,t) and H(r,t) results in losing the solutions to the equation of the electrodynamic state of the field (1.11) relative to the displacement u(r,t). Thus the area of application for the equations becomes much narrower.

The example of such a loss is a solution of equation (1.11) that leads to the appearance of waves with infinitely large amplitude, which in principle cannot be obtained as solution of Maxwell's equations.

5 Quantization of energy

In subsection 2.3 it was made apparent that the electrodynamic field intensity in case of oscillatory motion of charge carrier is described by relation (2.30), which is given below for convenience, (5.1),

$$E_{ed} = \frac{I_m}{2\pi\varepsilon_0\, r_0 c\, Y_1(\omega r_0/c)}\left(J_0\left(\omega\frac{r}{c}\right)\cos(\omega t) + Y_0\left(\omega\frac{r}{c}\right)\sin(\omega t)\right). \quad (5.1)$$

Let us analyze the changes in intensity E_{ed} regarding angular frequency ω as the only argument of function (5.1). Bessel function $Y_1(\omega r_0/c)$ in the denominator is a periodical function, which means that at angular frequency set of values $\{\omega_*\}$, which are the roots of the equation $Y_1(\omega r_0/c) = 0$, function (5.1) will have infinitely large values. From physical standpoint, it means infinitely large escalation of the intensity $E_{ed}(\omega)$ when angular frequency ω approaches the values ω_*, $E_{ed}(\omega) \mapsto \infty$ at $\omega \mapsto \omega_*$.

Putting r_0=1 mm, then as frequency is increased, the first root of the equation $Y_1(\omega r_0/c) = 0$ will be the frequency $\nu \approx 104.9$ GHz. Variation of the intensity $E_{ed}(\omega)$ of the electrodynamic field in the neighbourhood of this frequency is shown in Figure 8. Variation of the induced emf with time, $E_{ed}(t)$, at the frequency $\nu \approx 104.9$ GHz is plotted in Figure 9. The graphs have been plotted at the following parameters: Im = 10 A, r_0 =1 mm, r = 10m.

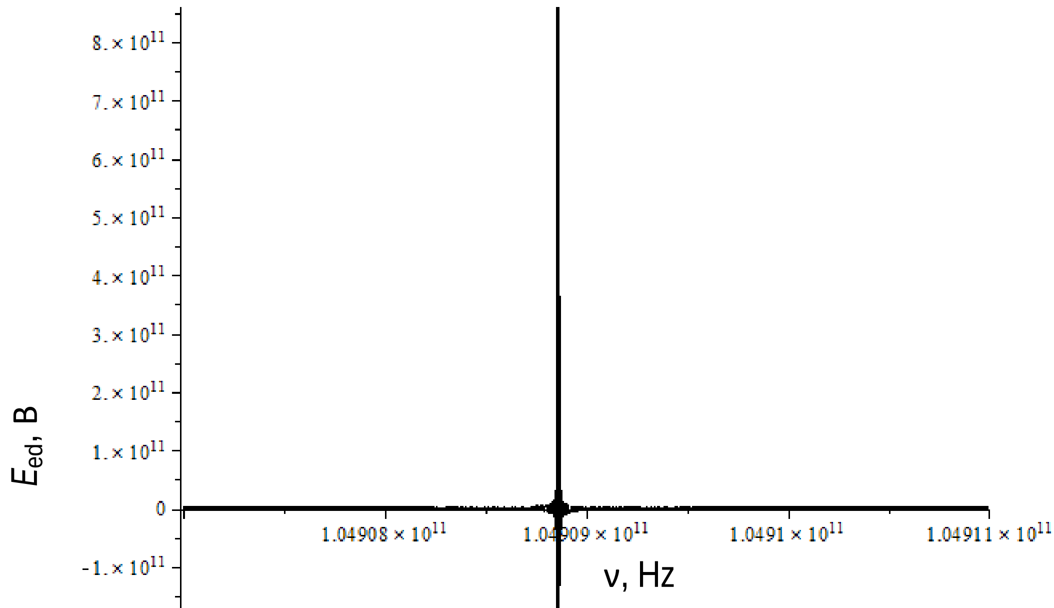

Figure 8

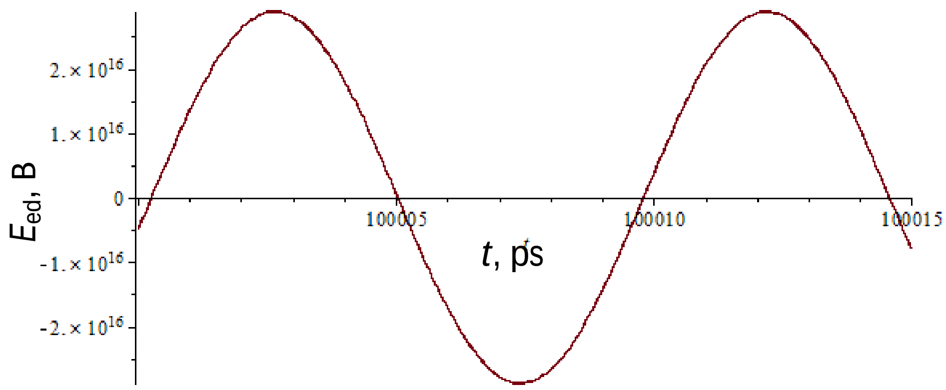

Figure 9

The values of the electrodynamic field intensity, as obvious from formula (5.1) and is illustrated by the graphs at Figures 8 and 9, may be so large that in case the wave falls on the surface of a solid body it is quite capable of causing the effect of photoemission.

For confirmation, let us compare the force f_{es} of electrostatic attraction between the electron and the nucleus in case of a hydrogen atom, and the force f_{ed} exerted on an electron by the electrodynamic field intensity, whose amplitude $E_{ed.m}$ we will consider equal ~ 3 10^{16} V/m (Figure 9). Then the average value of intensity for half-cycle will be $E_{mean\ ed} = E_{ed.m}/\pi \approx 10^{16}$. Under these conditions we have:

$$f_{es} = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{(1{,}6 \cdot 10^{-19})^2}{4\pi \cdot 8{,}85 \cdot 10^{-12} \cdot (5{,}3 \cdot 10^{-11})^2} = 8{,}2 \cdot 10^{-8} \text{ H}, \qquad (5.6)$$

$$f_{ed} = E_{mean\ ed}\, e = 10^{16} \cdot 1{,}6 \cdot 10^{-19} = 1{,}6 \cdot 10^{-3} \text{ H}. \qquad (5.7)$$

The electrostatic force turned out to five orders less than the force exerted on an electron by the electrodynamic field. The duration Δt of such action at the level of average value at a frequency of 104.909 GHz makes up Δt=2,17·10^{-12} s. This time is quite enough for the electron, when acted upon by the resultant of forces$f_{ed} - f_{es}$, ignoring the relativistic increase in mass, to develop a speed exceeding the speed of light:

$$v = (f_{ed}/m_e)t = \frac{1{,}6 \cdot 10^{-3} \cdot 2{,}17 \cdot 10^{-12}}{9{,}1 \cdot 10^{-31}} = 3{,}8 \cdot 10^{15}\ m/s\,. \qquad (5.8)$$

Ionization of an atom acted upon by the wave of electrodynamic intensity with the above parameters, turns out unavoidable.

Can we identify the burst of the electrodynamic field intensity, at single values of frequency from a set {ω*}, with a photon as a particle of electromagnetic radiation? Apparently, the answer is that a wave cannot be perceived as a particle, but the effect of such highly intensive electrodynamic field on the electrons of a solid body is quite capable of producing the very action that is now attributed to the photon.

To explain photoelectric effect, electromagnetic radiation was endowed with quantum properties, the notion of "quantum a energy" was constructed, and a virtual particle possessing this quantum was invented and called a photon. In the course of time, photoelectric effect has gained the status of one of the proofs of quantum properties of radiation, i.e. the proof of something that had been formerly invented.

The results outlined above provide quite a different explanation of such basic characteristic features of photoelectric effect as, for instance, the existence of some threshold frequency, in surpassing which the effect can appear, and its independence of the intensity of light. This explanation requires no references to virtual particles, let alone their acknowledgement as material objects capable of delivering work.

Conclusion

The results outlined in the article confirm the truth of G.R. Kirchhoff's words chosen for the epigraph. Electrodynamics, which up to now was based, if not exclusively, then for the most part, on Maxwell's postulates (equations), may be modeled with the methods used in continuum mechanics. Besides, the results obtained in so doing will provide theoretical grounding to both Maxwell's postulates and a number of other regularities found experimentally. Moreover, application of the theory of elasticity to the electric field provides a way for solving such problems as quantization of energy of electromagnetic radiation, which classical electrodynamics is unable to solve.