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The central problem in number theory and the mean value theorem of primes up to a given number *x*

***Key words:** Number theory, the Riemann Hypothesis, the number of prime numbers and composite numbers, units, the average number of primes.*

***Annotation:** I suggest a solution to the problem of the distribution of prime numbers in a series developed by me method thoroughly different from evidence of other mathematicians. So far this task has not been completed, because there is no guaranteed minimum amount of remaining member, and many of the proofs of the Prime Numbers Theorem (PNT) are very complex. I'm radically redefined the issue of distribution of prime numbers and found, in my opinion, the decision to order more precisely the previously known, and that is important, more simple and transparent.*

Introduction:

History of emergence of problems of prime numbers and its relationship with the Riemann Hypothesis. Mankind from time immemorial used arithmetic. In the arithmetic is the number 0 and the so-called natural number, which is the simplest of the logical sequence of positive integers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, etc.. It is not known exactly when, for the first time in the history of mathematics originated the concept of a prime number. The first researcher in number theory is probably the ancient Greek mathematician Euclid, who lived in III BC, he in his book IX of thearithmetic works "beginning" proved the infinity of prime number in natural number (1), (2, p. 34), (3, p. 17), (4, p. 19-20), (5, p. 1, 2 -18), (6, p. 177, 246 -247).

Method of determining the number of prime numbers up to a certain number of first invented by the ancient Greek mathematician, astronomer and philosopher Eratosthenes (III BC), the so-called "Eratosthenes (3, p. 100 -101), (8, p. 14), (9, p. 30 -33), (10, p. 32), (11, p. 12). This method is practical only for a small segment of the natural range of calculations, but it doesn't give any approximate formulas (functions) of prime numbers, nor a common picture of the distribution of prime numbers in a series. And because of the problem of distribution of prime numbers remained unclear until the end of the 18th century.

The great German mathematician Carl Friedrich Gauss (1777-1855) in December 1849, in response to the German astronomer J. F. Encke writes that he in 1792 or 1793 year, counting real number of primes $\pi(x)$ for every 1,000 number of natural numbers, revealed the dependence of $\pi(x)$, of the natural logarithm of this number (2, p .53 -54).

That is, Gauss conjectured that the number of primes$\pi(x)$ x can be expressed as the following expression:

$$\pi(x) \sim \frac{x}{lnx} \text{(2, p .53 -54)} \qquad (1.1)$$

At this time Gauss was 15 years! But the first published work related to The Prime Number Theorem, owned by Adrienne Marie Legendre. It's in the book "essays on the theory of numbers" (1798) suggested that

$$\pi(x) \sim \frac{x}{A \cdot lnx + B}; \qquad (1.2)$$

In a later edition of this book he clarified this assumption (to prove that he couldn't) thus:

$$\pi(x) = \frac{x}{lnx - B}; \qquad (1.3)$$

where: $B=1,08366$ (2, p .53 -54), (7, p. 255-263).

In 1823Abel in one letter called the Legendre function (1.3) "the most remarkable theorem in mathematics (5, p. 2-3). Of course, Abel was wrong! ... But for a few! Later I will prove that value is an essential element in the formula (1.3) and it is a variable. Gauss Legendre brought regardless of its function as the number of prime numbers, the so called "integral logarithm»

$$Li(x) = \int_2^x \frac{1}{lnt} dt; \qquad \text{(2, p. 113-117), (7, p. 256-257)} \qquad (1.4)$$

the $Li(x)$- is an integral expression $\frac{x}{ln\,x}$.

Any function that describes the number of prime numbers, gives error (difference) and because the relationship of $\pi(x)$ and $Li(x)$ write a formula:

$$\pi(x) = Li(x) + |R(x)|; \qquad \text{(7, p. 274-275)} \qquad (1.5)$$

where $R(x)$ is the error of calculation.

In 1852, in his work "On prime numbers" Russian mathematician ChebyshevP.l. proves that the upper and lower limits of the relationship $\frac{\pi(x)}{ln(x)}$ enclosed within 0.92129 and 1.10555, i.e.

$$0{,}92129 \frac{x}{lnx} < \pi(x) < 1{,}0555 \frac{x}{lnx}; \qquad \text{(7, p. 264)} \qquad (1.6)$$

In August 1859, the German mathematician Bernhard Riemann on the occasion of the adoption of its members-correspondents of the Berlin Academy of Sciences presented the work "On the Number of Prime Numbers Less Than a Given Quantity". (2, Prologue)

The Ryman proposed a new mathematical tool-so-called Zeta-function $\zeta(s)$:

$$\zeta(s) = \sum \frac{1}{n^s}; \qquad (1.7)$$

where: $s=\sigma+it$, $i = \sqrt{-1}$, and provided that the zeros of this function are closely linked to the distribution of prime numbers (2, p. 137-150, 324), (12, p. 5), (10, p. 321).

Speaking at the second International Congress of mathematicians in Paris in 1990, one of the greatest mathematical minds of the time, David Hilbert presented the report with 23 fundamental tasks for the near future. Number 8 was the Problem of prime numbers "by d. Hilbert said the following: ...to fully address the problems of the study of Riemann "on the number of primes less than a given magnitude" must first prove the crucial approval of the Riemann:

All non-trivial zeros of the Zeta-function have a real part equal to one second (2, p. 188 -190).

This is the well-known Riemann Hypothesis (RH) of the zeros of the Zeta-function. Today, this hypothesis has not been proven or disproven.
The Riemann Hypothesis is a gigantic scientific-research theme for many of the leading mathematical institutions.
Glue Mathematics Institute (founded in 1998 by Boston financier Landon T. Glue) installed a prize of one million United States dollars for the proof or disproof of each of the 7 of the so-called "Millennium Development Goals", including the Riemann Hypothesis (2, p. 351-354).
Building on the idea of Riemann, in 1896, the French mathematician Jacques Hadamardand Belgian mathematician Valle-Pussen independently proved the Prime Numbers Theorem (PNT):

$$\lim_{x\to\infty} \frac{\pi(x)}{\frac{x}{lnx}} = 1 \qquad (1.8)$$

And in 1949, a Danish mathematician A.Selberg andHungarian mathematician P. Erdesh proved the Prime Numbers Theorem (PNT) elementary methods (7, p. 273), (13), (14).

Now about the Riemann Hypothesis with prime numbers.
The calculated10^{13}zeros of the Zeta-function Riemann (1, p. 320). And no kontrprimera of the Riemann Hypothesis. But calculations have not yet confirmed the Riemann Hypothesis.

Home link the Riemann hypothesis and prime numbers is reflected in the following formula:

$$\pi(x) = Li(x) + O(\sqrt{x} ln\ x); \qquad (1.9)$$

where $\pi(x)$ is the number of primes;
$Li(x)$ - is an integral logarithm Gauss;
$O(\sqrt{x}\ ln\ x)$-error estimation calculation of $\pi(x)$.
If the Riemann Hypothesis is true, the right expression (1.9).This is the result of von Koch, 1901 (2, p. 242 -245, 381).

So, the mathematical world adopted Gaussian function *Li(x)* based on the number of primes up to a given number *x*. This was not the best direction in solving the problem of the distribution of prime numbers, since *Li(x)* is an integral product of the expression $\frac{x}{ln\ x}$, and the theoretical basis o *Li(x)* and the number of primes $\pi(x)$ is very difficult.

Technique.

1. Numerical-experimental analysis of functions $\frac{x}{ln\ x}$, $\frac{x}{ln\ x - B}$, $Li\ (x) = \int_2^x \frac{1}{lnt}\ dt$ and link them with the function of $\pi(x)$. The role of Legendre's constant value $B = 1,08366\ldots$

2. Identification of some ties with composite numbers, prime numbers.

3. Separation and composite numbers in the group numbers, and complex numbers on the blocks.

4. Calculate the average number of blocks and composite number based on the average number of primes *Es(x)* up to a particular number *x*.

5. The practical application of formula functions in the average number of primes *Es(x)* up to a particular number *x*.

The main part.

The Theorem: (Author: UshtenovYessenbek Riskulovich, certificate No. 636, dated May 22, 2012, issued by the Committee on the rights of intellectual property of the Ministry of Justice of the Republic of Kazakhstan).

The average number of primes $\pi(x)$ up to a given number x is equal to:

$$\pi(x) = \frac{x}{lnx-1}, \text{ if } x \to \infty. \quad (2.1)$$

Proof:

1. All known function $\frac{x}{lnx}$, $Li(x)$ and $\pi(x)$ and in many literature is their graphic representation. I cite Table 1 with the numerical values of these functions, which added a column with the values of the function $\frac{x}{lnx-B}$ (where $B = 1,08366\ldots$ - constant Legendre) and the graphic representation of these functions in Figure 1. It should be noted that the function $\frac{x}{lnx-B}$ gives a better approximation to π (x) function than the function Li(x) up to $x \approx 5000000$, but after that the x values. The best approximation gives $Li(x)$. And with increasing x it seems that $Li(x)$ becomes an indispensable feature on approximation to $\pi(x)$. Now take a look at Table 2 and 3, compiled by me.

##	x	$\pi(x)$	$Li(x)$	$\frac{x}{ln\,x}$	$\frac{x}{ln\,x - B}$
1	1 000	168	178	145	172
2	10 000	1 229	1 246	1 086	1231
3	50 000	5 133	5 167	4 621	5136
4	100 000	9 592	9 630	8 686	9 588
5	500 000	41 538	41 606	38 103	41 533
6	1 000 000	78 498	78 628	72 382	78 543
7	2 000 000	148 933	149 055	137 849	148 976
8	5 000 000	348 513	348 638	324 150	348 644
9	10 000 000	664 579	664 918	620 421	665 140
10	20 000 000	1 270 607	1 270 905	1 189 680	1 271 651
11	90 000 000	5 216 954	5 217 810	4 913 919	5 222 944
12	100 000 000	5 761 455	5 762 209	5 428 681	5 768 004
13	1 000 000 000	50 847 534	50 849 235	48 254 942	50 917 519

Table 1. $\pi\ (x)$ - is the real number of prime numbers up to x;
$Li\ (x)$ - is the number of prime numbers for integral logarithm Gauss;
$\frac{x}{\ln x}$ - is the number of prime numbers of Gaussian formula;
$\frac{x}{\ln x - B}$ - is the number of prime numbers of Legendres formula.

I introduced the Br (actual), defined as $Br = lnx - \frac{x}{\pi(x)}$ (taken from equality $\pi(x) = \frac{x}{lnx-Br}$), and showed her the numeric values in a separate column. According to these tables, it is clear that the amount of Br-variable, in contrast to the constant value in the Legendre. The Br is increased to its maximum 1,0962 ... when$x = 60000$, and then slowly, in leaps and

##	x	π (x)	ln x	$log_{pr}\,x=\frac{x}{\pi(x)}$	Br

bounds-decreasing, then increasing, on average, reduced to 1,0. While we do not know to what minimum value will decrease it. Based on these facts, we have the right to assign task to find the average theoretical value of Bt and its decision in the case, find the formula of the function of the average number of primes *Es(x)* up to a certain number of *x*:

$$E(x)=\frac{x}{\ln x - Bt} \qquad (2.2)$$

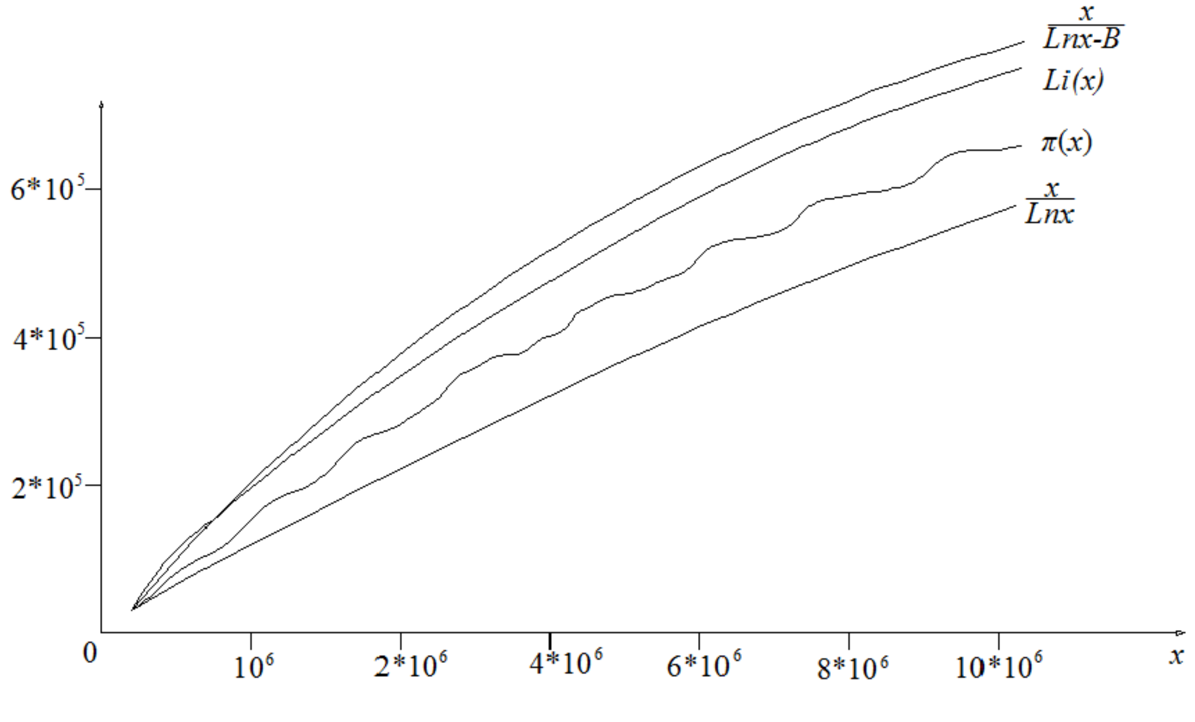

Figure 1. The graphic representation of functions $\frac{x}{Lnx\text{-}B}$, $Li(x)$, $\pi(x)$, $\frac{x}{Lnx}$.

1	100	25	4.6051…	4.0000…	0.6051…
2	1 000	168	6.9077…	5.9523…	0.9553…
9	10 000	1 229	9.2103…	8.1366…	1.0736…
4	100 000	9 592	11.5129…	10.4253…	1.0875…
5	1 000 000	78 498	13.8155…	12.7391…	1.0763…
6	10 000 000	664 579	16.1180…	15.0471…	1.0709…
7	100 000 000	5 761 455	18.4206…	17.3567…	1.0639…
8	1 000 000 000	50 847 534	20.7232…	19.6666…	1.0566…
9	10 000 000 000	455 052 511	23,0258…	21,9754…	1,0503…
10	100 000 000 000	4 118 054 813	25.3284…	24,2833…	1,0451…
11	1 000 000 000 000	37 607 912 018	27,6310…	25,5901…	1,0408…
12	10 000 000 000 000	346 065 536 839	29,9336…	28,8962…	1,0373…
13	100000 000 000 000	3204 941 750 802	32,2361…	31,2018…	1,0343…

Table 2. $\pi\ (x)$ - is the real number of prime numbers up to x;
$\ln x$ - is the natural logarithm of x;
$\log_{pr} x$ - logarithm of x to the base prime numbers;
Br - difference of the natural logarithm of a number x and logarithm on prime numbers to number x.

2. Lemma. If a natural number x of $\pi(x)$ is the prime numbers, so there is a composite number, the exact number of $s(x)$.

The converse is also true.

Proving of lemma.

According to lemma holds the following equality:

$$x=\pi(x)+s(x)+1, \quad (2.3)$$

where is the number 1 term is a unit that is neither simple nor the compound.

Use a sieve of Erotosfen to calculate the prime numbers up to a given number x.

Example:

1) $x = 10$, we find that $\pi(x) = 4$, if $s(x) = 10\text{-}4\text{-}1 = 5$, $s\ (x) = 5$;
2) $x = 100$, counting gives $\pi(x) = 25$, and $s(x) = 100\text{-}5\text{-}1 = 74$,
3) $x = 1000$, ... $\pi\ (x) = 168$, $s(x) = 1000\text{-}168\text{-}1 = 831$; and so on.

The lemma is proven.

This is an important lemma, that our study will give a picture of the ratio of simple and composite number in natural number for the task of finding one important variable. When analyzing the ratio of simple and compound numbers, we find that when $x = 10$ in creating composite numbers $s(x) = 5$ are all prime numbers $\pi(x) = 2, 3, 5, 7$, and only three of them: 2,3 and 5; in the example, $x = 100$, where there are $\pi(x) = 25$, the composite number in $s(x) = 74$ involved only 15 prime numbers, namely 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, while the remaining 10 prime numbers in this the process involved, but the example $x = 1000$ are 95 primes of 168: 2, 3, 5, 7, 11, ... 499.

If further investigations of this phenomenon, we confirm the fact of education their numbers up to x number of all the prime numbers less than or equal to *x/2*.

3. The question then arises: what is the function $\frac{x}{lnx}$ and the number formed by its?

The very notion of the natural logarithm ln x is the number of degrees, educated reason number *e = 2.71828182845 ...* . In the foundation of the number of degrees countain all simple and composite number to ∞.

In these examples it is:

if *x = 100,ln 100 = 4.6051...*, when *x = 1000, ln1000 = 6.9077…*, and so on.

And now number a series with natural numbers 1 to 100. Inclusive, imagine as blocks from simple or compound numbers:

number 1 and 2 - 1-th block;

number of 3.4 - 2-th block; and further

5.6 - 3-th block;

7, 8, 9, 10 - 4-th block;

11.12 - 5-th block;

13, 14, 15, 16 - 6-th block;

17.18 - 7-th block;

19, 20, 21, 22 - 8-j blok;

##	x	$ln\,x$	π(x)	$\log_{pr} x = \frac{x}{\pi(x)}$	Br
1	1 000	6,907755278…	168	5,952380952…	0,955374327…
2	10 000	9,210340371…	1 229	8,136696501…	1,073643871…
3	20 000	9,903487553…	2 262	8,84173298…	1,061754573…
4	30 000	10,30895266…	3 245	9,244992296…	1,063960365…
5	40 000	10,59663473…	4 203	9,517011658…	1,079623075…
6	50 000	10,81977828…	5 133	9,740892266…	1,078886019…
7	60 000	11,00209984…	6 057	9,905894007…	1,096205834…
8	70 000	11,15625052…	6 935	10,09372747…	1,062523052…
9	80 000	11,28978191…	7 837	10,20798775…	1,081794163…
10	90 000	11,40756495…	8 713	10,32939286…	1,078172088…
11	100 000	11,51292546…	9 592	10,42535446…	1,087571003…
12	200 000	12,20607265…	17 984	11,12099644…	1,085076204…
13	300 000	12,61153775…	25 997	11,53979305…	1,071744701…
14	400 000	12,89921983…	33 860	11,81334908…	1,085870742…
15	500 000	13,12236338…	41 538	12,03717078…	1,085192594…
16	600 000	13,30468493…	49 098	12,22045705…	1,084227889…
17	700 000	13,45883561…	56 543	12,37995862…	1,078876998…
18	800 000	13,59236701…	63 951	12,50957765…	1,082789361…
19	900 000	13,71015004…	71 274	12,62732553…	1,08282451…
20	1 000 000	13,81551056…	78 498	12,73917807…	1,07633249…
21	2 000 000	14,50865774…	148 933	13,42885727…	1,079800467…
22	3 000 000	14,91412285…	216 817	13,83655341…	1,07756944…

23	4 000 000	15,20180492…	283 146	14,12698749…	1,074817429…
24	5 000 000	15,42494847…	348 513	14,34666713…	1,078281345…
25	6 000 000	15,60727003…	412 849	14,53315861…	1,074111415…
26	7 000 000	15,76142071…	476 648	14,6858898…	1,07553091…
27	8 000 000	15,8949521…	539 777	14,82093531…	1,074016787…
28	9 000 000	16,01273514…	602 489	14,93803206…	1,074703072…
29	10 000 000	16,11809565…	664 579	15,04712006…	1,070975595…
30	100 000 000	18,42068074…	5 761 455	17,35672673…	1,063954014…
31	1 000 000 000	20,72326584…	50 847 534	19,66663713…	1,05662871…
32	10 000 000 000	23,02585093…	455 052 11	21,97548581…	1,050365116…
33	100 000 000 000	25,32843602…	4 118 054 813	24,28330961…	1,045126417…
34	1 000 000 000 000	27,63102112…	37 607 912 018	26,59014942…	1,040871694…
35	10000000000000	29,93360621…	346065 536839	28,89626078…	1,037345427…
36	100000000000000	32,23619130…	3204941750802	31,20181513…	1,034376175…

Table 3. $\pi(x)$ - is the real number of prime numbers up to x;
$ln\,x$ - is the natural logarithm of x;
$log_{pr}\,x$ - logarithm of x to the base prime numbers;
Br - difference of the natural logarithm of a number x and logarithm on prime numbers to number x.

23, 24, 25, 26, 27, 28 - 9-j blok;

……………………………..

and the last

97, 98, 99, 100 - 25-th block.

If one of these blocks to prime numbers, then only the composite number blocks:

The number of 4 - 1-th block;

number 6 - 2-th block;

number 8, 9, 10 - 3-th block; and further

12 - 4-th block;

14, 15, 16 - 5-th block;

18 - 6-th block;

20, 21, 22 - 7-th block;

24, 25, 26, 27, 28 - 8-th block; and so on ...

and the final block to $x = 100$:

98, 99, 100 - 24 St block.

So, $x = 100$ a total:

the number 1 (one) – is not prime number and not composite;

25–prime numbers; and

24- block their numbers.

So we got a major conclusion: the number of primes $\pi(x)$ is the number of blocks b_{sr}+ 1, i.e. equality holds:

$$\pi(x) = b_{sr} + 1. \quad (2.4)$$

Therefore, the average number of primes *Es(x)* can be calculated indirectly through the mean theoretical number of blocks of the composite number b_{st}:

$$Es(x) = b_{st} + 1. \quad (2.5)$$

But, as we noted above, in the creation of the composite numbers *s(x)* to *x = 100* are only 15 prime numbers, and *x = 1000*-95 total are prime numbers, i.e., a composite number *s(x)* to *x* are all prime numbers up to the number *x/2*. Denote the number of primes, involved in the formation of the composite number to number x through $p_e$and composite number to the number of blocks of numbers x throughb_{sr}.

Now look at table 4. According to this table shows that the numerical value of the expression $\frac{x}{\ln x}$is between the numerical values $p_e$andb_{sr}, that is the expression $\frac{x}{\ln x}$is a numerical value between the actual average number of primes up to *x/2* numbers and the number of blocks of the composite number b_{sr}.Means,*lnx* being equal to e = 2.71828182845 ... to number *x*, actually reflects some number. What is this number? Imagine a natural number 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ... 32. 360. in the following form:

$K_1^0,\ K_2^1,\ K_3^1,\ K_4^2,\ K_5^1,\ K_6^2,\ K_7^1,\ K_8^3,\ K_9^2,\ K_{10}^2,\ K_{11}^1, K_{12}^3,\ \ldots K_{32}^5, \ldots K_{360}^6, \ldots,$

where the $K_1^0=1$, $K_2^1=2$, $K_3^1 = 3, \ldots,$ $K_{12}^3 = 12 = 2\cdot 2\cdot 3, \ldots,$ $K_{32}^5 = 32 = 2\cdot 2\cdot 2\cdot 2\cdot 2, \ldots,$ $K_{360}^6 = 2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 5, \ldots$ and so on.

It is clear that in the last row number reflects the number of factors of a composite number, and number 1 in the level means that it is a prime number. From these considerations, it can be concluded that *ln x* is the average of the number of factors all Prime and composite number to number *x*, and $\frac{x}{\ln x}$reflects the average number of blocks and composite numbers from number 1 to number x. But as we have previously pointed out that prime numbers to number *x* can be divided into 2 different groups according to their functional properties: one group is involved in the creation of the composite number to number *x*and the other is not, because the simple numbers greater than *x/2*, may not have terms of prime numbers and therefore cannot form a composite number to the number *x*.

Therefore, to calculate logarithmic expression through the average number of factors (degrees) all blocks of the composite number to number *x*, denoted it as the *x*, you must $\log_{pr} x$of base *e = 2.71828182845 ...* subtract the base composite numbers to x and then the expression $\frac{x}{\log_{pr} x}$where pr is the base for all prime numbers up to *x* = ∞, the number will be equal to the number of blocks of the composite number and, consequently, the number of prime numbers. As we have seen from the calculation table 3 actual $log_{pr}\, x$-variable and, therefore, the founding of pr is too variable. But the mechanism for this reason pr us is unknown and probably very complex. To obtain the theoretical value of $\ln x log_{pt}\, x x$ subtract part of composite numbers, we have the above named *B* (instead of constant Legendre in which he was able to give theoretical justification (11, p. 260) see table 1). On the basis of these judgments can record:

$$log_{pt}\, x = \ln x\text{-}Bt, \quad (2.6)$$

4. Now comes the question: what's the value of *Bt* and how to calculate it?

From tables 2 and 3 show that Bt is part of *lnx*, and *lnx* the reason is the comprised $e = (1 + \frac{1}{x})^x$ of simple and composite numbers and due to this fact, *ln x* is the sum of the degrees of the composite number $log_{pt}\, x$, educated prime numbers and degrees of prime numbers, composite numbers, *Bt*:

$$ln\, x = log_{pt}\, x + Bt, \qquad (2.7)$$

##	x	$\pi(x)$	$\frac{x}{ln\, x}$	p_e	b_{sr}
1	100	25	21,7…	15	24
2	150	35	29,9…	21	34
3	200	46	37,7…	25	45
4	250	52	45,2…	30	51
5	300	62	52,5…	35	61
6	350	70	59,7…	40	69
7	400	78	66,7…	46	77
8	450	87	73,6…	48	86
9	500	95	80,4…	52	94
10	550	101	87,1…	58	100
11	600	109	93,7…	62	108
12	650	118	100,3…	66	117
13	700	125	106,8…	70	124
14	750	132	113,2…	74	131
15	800	139	119,6…	78	138
16	850	146	126,0…	82	145
17	900	154	132,3…	87	153
18	950	161	138,5…	91	160
19	1 000	168	144,7…	95	167

Table 4. $\pi(x)$ - is the real number of prime numbers up to *x*;
$\frac{x}{\ln x}$ - is the number of prime numbers of Gaussian formula;
p_e - is the number of prime numbers involved in the formation of the composite number to number *x*;
b_{sr} - composite number, number of blocks and the number of *x*.

Therefore, you must subtract *x ln*, formed by composite numbers:

$$Bt = ln\left(1 + \frac{1}{s(x)}\right)^x = ln\left(1 + \frac{1}{x-\pi(x)}\right)^x, \qquad (2.8)$$

and then the

$$log_{pt}\, x = ln\, x - Bt = ln\, x - ln\left(1 + \frac{1}{x-\pi(x)}\right)^x \qquad (2.9)$$

So, the average theoretical number of blocks of the composite number is equal to:

$$b_{st} = \frac{x}{log_{pt}\, x} = \frac{x}{ln\, x - Bt} = \frac{x}{ln\, x - ln\left(1+\frac{1}{x-\pi(x)}\right)^x}; \qquad (2.10)$$

And, ignoring the summand 1 in latest formula (2.5), finally the mean theoretical number of prime numbers is:

$$Es(x) = \frac{x}{ln\, x - ln\left(1+\frac{1}{x-\pi(x)}\right)^x} \qquad (2.11)$$

Based on proven by Euler's theorem: $\frac{\pi(x)}{x} \to 0$, when $x \to \infty$ [6; 254] can assert that the $lim(1+\frac{1}{x-\pi(x)})^x = e$, when $x \to \infty$; (e = 2.71828182845 ...) and $lim ln\left(1+\frac{1}{x-\pi(x)}\right)^x = 1$, when $x \to \infty$.

and finally the number of prime numbers is:

$$\pi(x) = \frac{x}{lnx-1}, \quad \text{when } x \to \infty \qquad (2.12)$$

The theorem is proved.

So, the formula functions in the average number of primes *Es(x)* up to a given number *x* is as follows:

$$Es(x) = \frac{x}{ln\,x - ln\left(1+\frac{1}{x-\pi(x)}\right)^x} = \frac{x}{ln\frac{x}{(1+\frac{1}{x-\pi(x)})^x}}. \qquad (2.13)$$

5. Then comes the question of the practical side of the calculation of the average number of *Es(x):* is it possible to calculate the amount that is on both sides of the equality? ... Like this is nonsense! But the experimental calculations show that the ratio of the average number of Prime and composite number is the average of the numbers in such a way that a few transactions is a balance of the left and right sides.

I will explain this process: in the formula $Es(x) = \frac{x}{ln\frac{x}{(1+\frac{1}{x-\pi(x)})^x}}$ we do not know in advance the number π (x), and therefore, in the first operation of the *Es(x)* to temporarily replace the $Es_1(x)$, and *π(x)* replace the *x/lnx*, knowing that the number x is prime numbers more than *x/lnx*. Next. To verify that the received result $Es_1(x)$ produce a second operation, substituting in the formula (2.12) $Es_1(x)$ to the $Es_2(x)$, and in the formula λ_2 we replace *x/lnx* on $Es_1(x)$ and so do several operations on calculation of *Es(x)*. The fourth and subsequent operations, we are convinced that the numerical value of the *Es(x)* has stabilized, and that the final result could take $Es_3(x)$. By the way, this is one of the most remarkable properties of the ratio of simple and composite number shows the mutual influence on each other, and also proves the infinite number of primes.

So, the average number of primes *Es(x)* to be calculated by the following procedure:

$Es_1(x) = \frac{x}{ln\frac{x}{\lambda_1}}$, where $\lambda_1=\left(1+\frac{1}{x-x/\,ln\,x}\right)^x$, if the $\left(1+\frac{1}{x-x/\,ln\,x}\right)^x$ < 2.99010913838,

otherwise the λ_1=5,98021827675 - $\left(1+\frac{1}{x-x/\,ln\,x}\right)^x$, (2.14)

$Es_2(x) = \frac{x}{ln\frac{x}{\lambda_2}}$, where $\lambda_2=\left(1+\frac{1}{x-Es_1(x)}\right)^x$, if $\left(1+\frac{1}{x-Es_1(x)}\right)^x$ < 2.99010913838,

otherwise the λ_2=5,98021827675 - $\left(1+\frac{1}{x-Es_1(x)}\right)^x$, (2.15)

$Es(x) = Es_3(x) = \frac{x}{\ln\frac{x}{\lambda_3}}$, where $\lambda_3=\left(1+\frac{1}{x-Es_2(x)}\right)^x$, if $\left(1+\frac{1}{x-Es_2(x)}\right)^x$ < 2.99010913838, otherwise the λ_3=5,98021827675 - $\left(1+\frac{1}{x-Es_2(x)}\right)^x$, (2.16)

The calculations *Es(x)*see Table 5 and Table 6.

Now an explanation why there were $\lambda_1, \lambda_2, \lambda_3$ is and why the should not exceed some number (call it *se*) *se* = 2.99010913838 ...

Perform calculations the following values: the number of degrees of evidence of actual and theoretical parts of the natural logarithm - $e^{Br} = e^{\ln x - \frac{x}{\pi(x)}}$ from formula $Br = \ln x - \frac{x}{\pi(x)}$ and $e^{Bt} = \left(1 + \frac{1}{s(x)}\right)^x$ from formula $Bt = \ln\left(1 + \frac{1}{s(x)}\right)^x$, as

##	*x*	*π(x)*	*Es(x)*	*Es(x)-π(x)*	k_2
1	100	25	26	+1	0,00000000001…
2	150	35	36	+1	0,00000000001…
3	200	46	45	-1	0,00000000001…
4	250	52	54	+2	0,1255…
5	300	62	62	0	0
6	350	70	71	+1	0,00000000001…
7	400	78	79	+1	0,00000000001…
8	450	87	87	0	0
9	500	95	95	0	0
10	550	101	102	+1	0,00000000001…
11	600	109	110	+1	0,00000000001…
12	650	118	118	0	0
13	700	125	125	0	0
14	750	132	133	+1	0,00000000001…
15	800	139	140	+1	0,00000000001…
16	850	146	147	+1	0,00000000001…
17	900	154	154	0	0
18	950	161	161	0	0
19	1 000	168	169	+1	0,00000000001…

Table 5. *π (x)* - is the real number of prime numbers up to *x*;
Es(x) - is the number of prime numbers of Ushtenovs formula ;
Es(x) - π (x) - is the absolute difference between the *Es(x)* and *π(x);*
k_2 - degree in the expression x^{k_2} = *Es(x) - π(x)* .

Well as the arithmetic mean values se = $(e_o^{Br} + e_o^{Bt})/2$, where $e_o^{Br} = (e_1^{Br} + e_2^{Br} + \ldots + e_n^{Br})/n$, $e_0^{Bt} = (e_1^{Bt} + e_2^{Bt} + \ldots + e_n^{Bt})/n$. It is clear that e_o^{Br} and e_0^{Bt} - arithmetic values and are determined by the interval n, where is the maximum value. On the basis of these calculations will provide a graphic image e^{Br}, e^{Bt}*se* and2*se*. See the Figure 2 and Table 7.

Table 7 take the estimated value, the maximum value of the column 2.99010913838 se = Q. In accordance with Figure 2 you can see that

##	*x*	*π(x)*	*Li(x)*	*Li(x)-π(x)*	k_1	*Es(x)*	*Es(x)-π(x)*	k_2
1	1 000	168	178	+10	0,3333…	169	+1	0,000000001…
2	2 000	303				303	0	0
3	3 000	430				429	-1	0,000000001…
4	4 000	550				550	0	0
5	5 000	669				668	-1	0,000000001…
6	6 000	783				783	0	0

7	7 000	900				896	-4	0,1565…
8	8 000	1 007				1007	0	0
9	9 000	1 117				1117	0	0
10	10 000	1 229	1 246	+17	0,3076…	1225	-4	0,1505…
11	20 000	2 262				2262	0	0
12	30 000	3 245				3247	+2	0,0672…
13	40 000	4 203				4200	-3	0,1036…
14	50 000	5 133	5 167	+34	0,3259…	5131	-2	0,0640…
15	60 000	6 057				6046	-11	0,2179…
16	70 000	6 935				6947	+12	0,2227…
17	80 000	7 837				7837	0	0
18	90 000	8 713				8717	+4	0,1215…
19	100 000	9 592	9 630	+38	0,3159…	9589	-3	0,0954…
20	200 000	17 987				17995	+8	0,1703…
21	300 000	25 997				26 050	+53	0,3148…
22	400 000	33 860				33 879	+19	0,2282…
23	500 000	41 538	41 606	+68	0,3215…	41 557	+19	0,2243…
24	600 000	49 098				49 118	+20	0,2251…
25	700 000	56 543				56 584	+41	0,2759…
26	800 000	63 951				63 972	+21	0,2239…
27	900 000	71 274				71 292	+18	0,2108…
28	1 000 000	78 498	78 628	+130	0,3523…	78 553	+55	0,2900…
29	2 000 000	148 933	149 055	+122	0,3311…	148 940	+7	0,1341…
30	3 000 000	216 817				216 822	+5	0,1079…
31	4 000 000	283 146				283 173	+27	0,2168…
32	5 000 000	348 513	348 638	+125	0,3130…	348 432	-81	0,2848…
33	6 000 000	412 849				412 843	-6	0,1148…
34	7 000 000	476 648				476 568	-80	0,2780…
35	8 000 000	539 777				539 716	-61	0,2586…
36	9 000 000	602 489				602 369	-120	0,2989…
37	10 000 000	664 579	664 918	+339	0,3614…	664 588	+9	0,1363…

Table 6. *π (x)* - is the real number of prime numbers up to *x*;
Li(x) - is the number of primes for Gauss's integral logarithm;
Li(x) - π(x) - is the absolute difference of *Li(x)* and *π(x)*;
k_1 - degree in the expression x^{k_1} = *Li(x) - π(x)* ;
Es(x) - is the number of prime numbers formula Ushtenov E.R.
Es(x) - π (x) - is the absolute difference between the *Es(x)* and *π(x)*
k_2 - degree in the expression x^{k_2} = *Es(x) - π(x)* .

Line true e_0^{Br} is almost a mirror image of the theoretical values of e_0^{Bt} to the horizontal line *se*. The reason for this phenomenon is "glut" of prime numbers at the beginning of the series, the natural formation which is much faster than education odd composite numbers, and this fact affects the debt over and, consequently, the value of λ_1, λ_2, and λ_3, (and theoretical), greater than the number of *se* = 2.99010913838 ... . Because the calculations must be artificially understate it: *2se* -λ_n to threshold values for 2.99010913838 ... if λ_n> 2.99010913838…, (5.98021827675 .../2 = 2.99010913838 ...). This is confirmed by the data Calculation bases *2se* and tables 2 and 3 and where you can see that the *Br* the increases, then decreases, but the average is growing and reaches a maximum value of Br = 1.0962 ... when x = 60 000, and continue to be unevenly, stepwise decreases of 1.0 up to x = 100 000. Practical calculations show (see. drive with the calculations of primes up to x = 10 000 000) that "saturation" retreating at the first operation calculations $Es(x)$ if x = 97900, second operation - if x = 290200 and the third is at x = 292800. After the value x = 292800 is the period of sustained decrease of $e^{Bt} = \left(1+\frac{1}{s(x)}\right)^x$, se = 2.99010913838 ... the number e = 2.71828182845 ... .

##	Column	Sum	Number of items	2se	Se
1	K	2483.33787785330	417	5.95524670983	2.97762335000
2	L	807.28359320841	135	5.97987846815	2.98993923400
3	M	1439.50079466998	241	5.97303234430	2.98651617200
4	N	496.13018145833	83	5.97747206630	2.98873603288
5	O	1987.00941988903	333	5.96699525492	2.98349762746
6	P	2408.34574150796	404	5.96125183541	2.98062591770
7	Q	598.02182767521	100	5.98021827675	2.99010913838
8	R	400.14143546049	67	5.97226023075	2.98613011538
9	S	585.99880359399	98	5.97957962851	2.98978981425

Table 7. $2se$ -the amount of the theoretical and practical values of e_o^{Br} and e_0^{Bt}; se -is the arithmetic mean e_o^{Br} and e_0^{Bt}.

And you can logically conclude that the process of reducing that number already is irreversible and it is obvious that $lim\left(1+\frac{1}{x-Es(x)}\right)^x = e$, when $x \to \infty$ and the limBt =1 , when $x \to \infty$.

It is likely that the more accurate calculations, you can get a se = 3.00 ...

In the accompanying article content with the calculations the average number of primes $Es(x)$ payments are limited to the number x = 10000000. The fact of the matter is that the calculations show that if $x >$ 10 000 000 $Es(x)$ are lower than they should be. This is due to the fact that during the construction of the number to the power of x, $1+\frac{1}{x-Es_n(x)}$ is a number with each subsequent operation has more decimal digits after the comma integer. For example, if x = 1 000 000 000 number $\left(1+\frac{1}{x-x/\ln x}\right)^x$ = 2.8596565834168656156005309067946 ..., but the computer in a table cell in Microsoft Office Excel 2007 displays only 15 digits: 2.85965650274005, producing inaccuracies in the calculation, and discarding all other numbers, although in reality this number contains 9 000 000 000 digits after the decimal point.

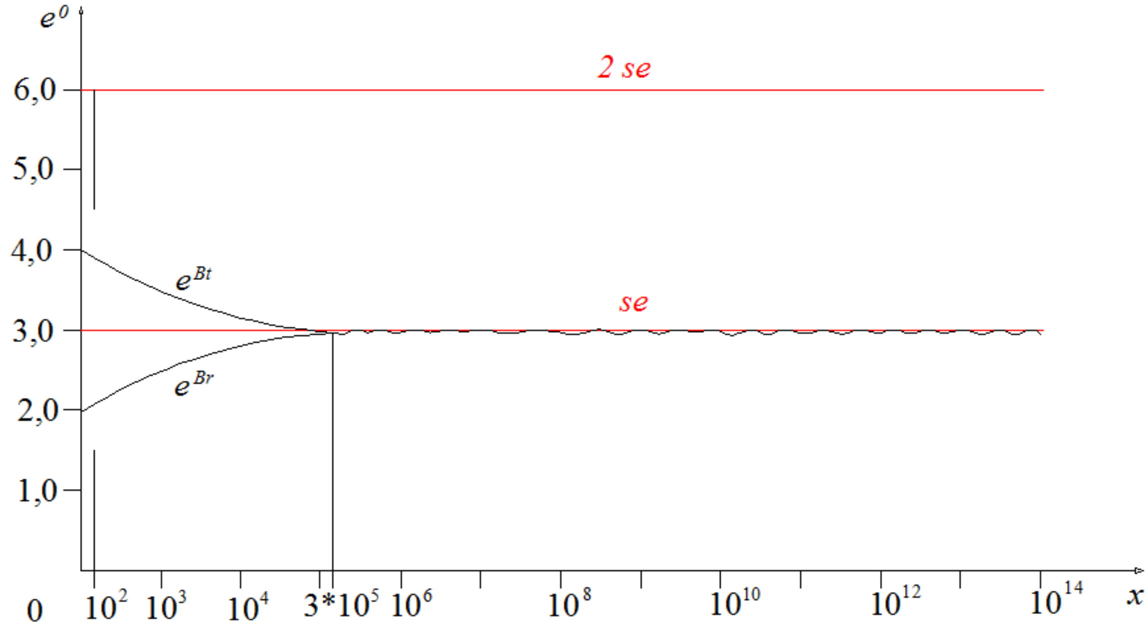

Figure 2. The graphic representation of value e^{Bt}, e^{Br}, se.

Therefore, the number of $\left(1+\frac{1}{x-Es(x)}\right)^{x}$, if x over 10 000 000 better accept values that are less than they should be and, respectively, the true value of the*Es(x)*. The second reason to calculate the natural logarithm of x/λ_n when base e is a limited number of computer program, when in fact it is transcedentnym and has an infinite number of digits after the decimal point, and because when we get a smaller numerical value *lnx* than must be actually ...

On the basis of these facts show the calculations of the number of prime numbers if x over 10000000 is meaningless because they undervalued the numeric value.

Based on a proven theorem we conclude that:

$$\pi(x) = Es(x) + |r(x)|; \qquad (2.17)$$

where *r(x)*- is the residual term (error of calculation).

Conclusion.

According to a theorem, a function of *Es(x)* is whole, positive and continuous, because from the numbers of natural row the defined numeral value *Es(x)* corresponds every argument of *x*. *Es(x)*, in turn, there is a numeral mean value from actual $\pi(x)$ - it is confirmed by calculations(see tables 5 and 6) and a next chart takes place on a Figure 3, where *Es(x)* is a middle line in graphic arts $\pi(x)$. On the basis of these results, we can assert that equality $\pi(x) = Es(x) + |r(x)|$ on an order more precisely, than $\pi(x)=Li(x) + |R(x)|$, and accordingly $|r(x)|<|R(x)|$.

A central problem of theory of numbers is distribution of prime numbers in a natural row was pre-condition of birth of Hypothesis of Riemann (12, p. 4). But also Hypothesis of Riemann did not decide this task to a full degree, that is why however she is yet well-proven.

I think that the problem of distribution of prime numbers, decided by me by elementary methods in part of receipt of function describing the quantitative height of prime numbers with the height of natural row, has the advantage in simplicity and practicality as compared to other functions, and also the theoretical base of function of *Es(x)* is brought. The task of determination of theoretical size of remaining member of *r(x)* remains unsolved in the formula $\pi(x) = Es(x) + |r(x)|$. But this problem I left for scopes this work, because it is related to Hypothesis of Riemann and many tasks of non-standard character will appear during the decision of her, requiring new approaches.

Gratitude to:
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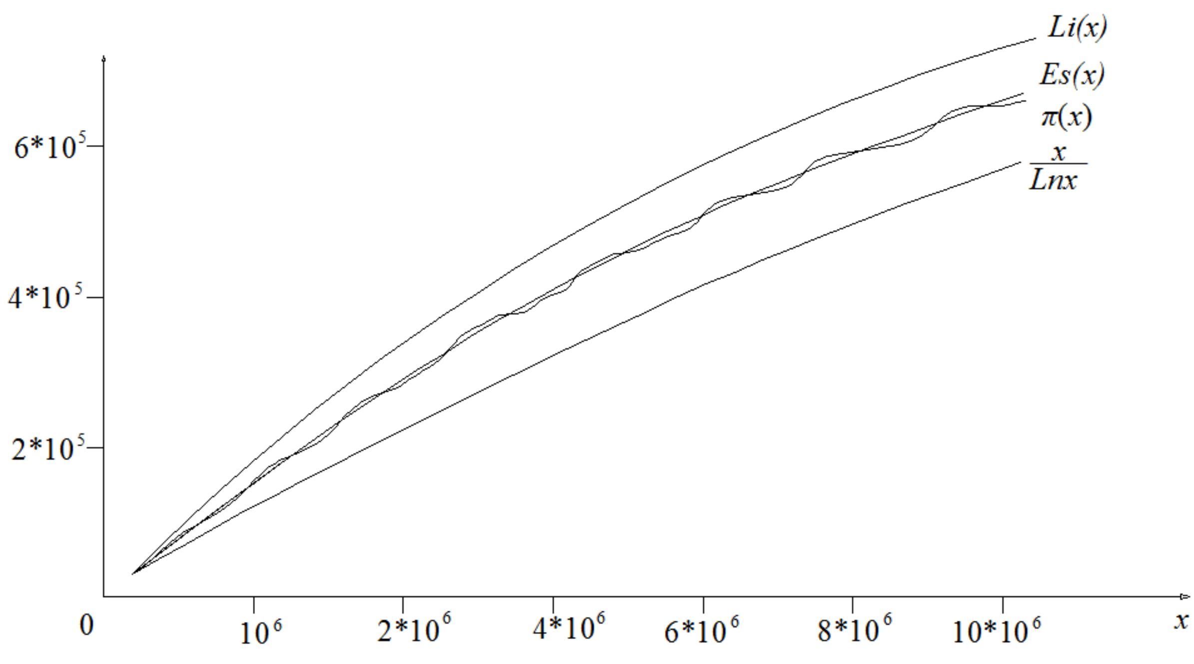

Figure 3. The graphic representation of functions $Li(x)$, $Es(x)$, $\pi(x)$, $\frac{x}{Lnx}$.

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